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Modern trade theory for CGE modellers: the Armington, Krugman and Melitz models

by

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Trade in CGE modelling

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Pre 1970s Heckscher-Ohlin - imports and domestic products identical, constant returns to scale, perfect competition
→ big gains from trade but unrealistic specialization

1970s- Armington - import/domestic imperfect substitution (variety at country level), constant returns to scale, perfect competition
→ big negative terms-of-trade effects from cutting tariffs even for small countries, often dominate positive efficiency gain

1980s- Krugman - variety at firm level rather than country level, increasing returns to scale, monopolistic competition among identical firms
→ still get big negative terms-of-trade effects but potential extra welfare from additional variety and increasing returns to scale

2003- Melitz - variety at firm level, increasing returns to scale, monopolistic competition among firms with different productivity
→ still get big negative terms-of-trade effects but potential extra welfare from additional variety, increasing returns to scale, and pro-trade productivity effect

Introduction

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- Derive the Armington, Krugman and Melitz models of trade as special cases of a general model.
- Examine optimality properties of Melitz
- Look at the Balistreri-Rutherford decomposition algorithm: solves Melitz general equilibrium by iterating between Melitz sectoral models and an Armington general equilibrium model
- Set up numerical Melitz model
- Demonstrate that Melitz welfare results can be decomposed into Armington effects
- Show that Melitz results look like Armington results with a higher substitution elasticity

Encompassing model: demand

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Country j's demand for varieties of widgets from all countries

People in country j choose Q_{sj} and Q_{ksj} to minimize:

$$\sum_s \sum_{k \in S(s,j)} Q_{ksj} P_{ksj}$$

subject to

$$Q_{sj} = \left(\sum_{k \in S(s,j)} \gamma_{ksj} Q_{ksj}^{-\rho} \right)^{-1/\rho} \qquad (4)$$

and

$$Q_j = \left(\sum_s \delta_{sj} Q_{sj}^{-\rho} \right)^{-1/\rho}$$

Encompassing model: demand functions

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$$Q_{ksj} = Q_j \left(\delta_{sj}\gamma_{ksj}\right)^{\sigma} \left(\frac{P_j}{P_{ksj}}\right)^{\sigma} \quad \text{and} \qquad (3)$$

$$P_j = \left(\sum_s \sum_{k \in S(s,j)} \left(\delta_{sj}\gamma_{ksj}\right)^{\sigma} P_{ksj}^{1-\sigma} \right)^{1/(1-\sigma)} \qquad (2)$$

Encompassing model: profits

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Contribution to profits of firm k,s from sales to j

$$\Pi_{ksj} = P_{ksj}Q_{ksj} - \left(\frac{W_s T_{sj}}{\Phi_{ks}}\right)Q_{ksj} - F_{sj}W_s \qquad (5)$$

Industry profits in country s

$$\Pi_s = \sum_j \sum_{k \in S(s,j)} \Pi_{ksj} - N_s H_s W_s \qquad (6)$$

Encompassing model: prices

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$$\mathbf{P}_{\mathbf{ksj}} = \left(\frac{\mathbf{W}_{\mathbf{s}} \mathrm{T}_{\mathbf{sj}}}{\Phi_{\mathbf{ks}}} \right) \left(\frac{\eta}{1+\eta} \right) , \quad \eta < -1 \qquad \textbf{(1)}$$

Lerner mark-up rule: η is the perceived elasticity of demand

Encompassing model: widget employment in country s

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$$L_s = \sum_j \sum_{k \in S(s,j)} \frac{Q_{ksj}}{\Phi_{ksj}} + \sum_j N_{sj} F_{sj} + N_s H_s \qquad (7)$$

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Encompassing model: fundamental equations for widget sector

$$\mathbf{P}_{ksj} = \left(\frac{\mathbf{W}_s T_{sj}}{\Phi_{ks}}\right)\left(\frac{\eta}{1+\eta}\right), \quad \eta < -1 \tag{1}$$

$$\mathbf{P}_j = \left(\sum_s \sum_{k \in S(s,j)} \left(\delta_{sj}\gamma_{ksj}\right)^{\sigma} \mathbf{P}_{ksj}^{1-\sigma}\right)^{1/(1-\sigma)} \tag{2}$$

$$\mathbf{Q}_{ksj} = \mathbf{Q}_j \left(\delta_{sj}\gamma_{ksj}\right)^{\sigma} \left(\frac{\mathbf{P}_j}{\mathbf{P}_{ksj}}\right)^{\sigma} \tag{3}$$

$$\mathbf{Q}_{sj} = \left(\sum_{k \in S(s,j)} \gamma_{ksj} \mathbf{Q}_{ksj}^{-\rho}\right)^{-1/\rho} \tag{4}$$

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Encompassing model: fundamental equations for widget sector

$$\Pi_{ksj} = \mathbf{P}_{ksj}\mathbf{Q}_{ksj} - \left(\frac{\mathbf{W}_s \mathrm{T}_{sj}}{\Phi_{ks}}\right)\mathbf{Q}_{ksj} - \mathbf{F}_{sj}\mathbf{W}_s \qquad (5)$$

$$\Pi_s = \sum_j \sum_{k \in S(s,j)} \Pi_{ksj} - \mathbf{N}_s \mathbf{H}_s \mathbf{W}_s \qquad (6)$$

$$\mathbf{L}_s = \sum_j \sum_{k \in S(s,j)} \frac{\mathbf{Q}_{ksj}}{\Phi_{ksj}} + \sum_j \mathbf{N}_{sj}\mathbf{F}_{sj} + \mathbf{N}_s \mathbf{H}_s \qquad (7)$$

Armington model

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F_{sj} = 0 and H_s = 0 (no fixed costs);

$\eta = -\infty$ (producers in country s are competitive);

$\Phi_{ks} = \Phi_s$ for all k (there is no difference in productivity across firms in country s);

γ_{ksj} = 1 for all k (preferences of j are symmetrical across firms on the sj link);

N_s is exogenous, assume N_s =1

Krugman model

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$F_{sj} = 0$ but $H_s > 0$ (fixed costs to produce any variety in country s);

$\eta = -\sigma$ (perceived elasticity is actual elasticity);

$\Phi_{ks} = \Phi_s$ for all k (there is no difference in productivity across firms in country s);

$\gamma_{ksj} = 1$ for all k (preferences of j are symmetrical across firms on the sj link);

N_s is endogenous: need an extra restriction relative to Armington

S(s,j) contains all N_s firms

Melitz model

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$F_{sj} > 0$ and $H_s > 0$;

$\eta = -\sigma$ (perceived elasticity is actual elasticity);

Φ_{ks} differs across k;

$\gamma_{ksj} = 1$ for all k (preferences of j are symmetrical across firms on the sj link);

N_s is endogenous: need an extra restriction relative to Armington

Proportion of firms (N_{sj}/N_s) in S(s,j) is endogenous: need extra restrictions relative to Armington and Krugman

Price of typical variety on sj link
(getting rid of the k dimension)

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$$P_{ksj} = \left(\frac{W_s T_{sj}}{\Phi_{ks}}\right)\left(\frac{\eta}{1+\eta}\right) \quad , \quad \eta < -1 \qquad (1)$$

Armington

$$P_{\bullet sj} = \left(\frac{W_s T_{sj}}{\Phi_s}\right)$$

Krugman

$$P_{\bullet sj} = \left(\frac{W_s T_{sj}}{\Phi_s}\right)\left(\frac{\sigma}{\sigma-1}\right)$$

Melitz

$$P_{\bullet sj} = \left(\frac{W_s T_{sj}}{\Phi_{ave(s,j)}}\right)\left(\frac{\sigma}{\sigma-1}\right)$$

Average price in country j
(getting rid of the k dimension)

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$$\mathbf{P_j} = \left(\sum_s \sum_{k \in S(s,j)} \left(\delta_{sj} \gamma_{ksj} \right)^{\sigma} P_{ksj}^{1-\sigma} \right)^{1/(1-\sigma)} \qquad \mathbf{(2)}$$

Armington

$$P_j = \left(\sum_s \delta_{sj}{}^{\sigma} P_{\bullet sj}^{1-\sigma} \right)^{1/(1-\sigma)}$$

Krugman

$$P_j = \left(\sum_s N_s \delta_{sj}{}^{\sigma} P_{\bullet sj}^{1-\sigma} \right)^{1/(1-\sigma)}$$

Melitz

$$P_j = \left(\sum_s N_{sj} \delta_{sj}{}^{\sigma} P_{\bullet sj}^{1-\sigma} \right)^{1/(1-\sigma)}$$

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J's demand for typical variety from s

(getting rid of the k dimension)

$$Q_{ksj} = Q_j \left(\delta_{sj}\gamma_{ksj}\right)^{\sigma} \left(\frac{P_j}{P_{ksj}}\right)^{\sigma} \qquad (3)$$

Armington

$$Q_{\bullet sj} = Q_j \delta_{sj}^{\ \sigma} \left(\frac{P_j}{P_{\bullet sj}}\right)^{\sigma}$$

Krugman

$$Q_{\bullet sj} = Q_j \delta_{sj}^{\ \sigma} \left(\frac{P_j}{P_{\bullet sj}}\right)^{\sigma}$$

Melitz

$$Q_{\bullet sj} = Q_j \delta_{sj}^{\ \sigma} \left(\frac{P_j}{P_{\bullet sj}}\right)^{\sigma}$$

Aggregate flows from s to j
(getting rid of the k dimension)

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$$\mathbf{Q_{sj}} = \left(\sum_{\mathbf{k \in S(s,j)}} \gamma_{\mathbf{ksj}} \mathbf{Q_{ksj}^{-\rho}} \right)^{-1/\rho} \qquad \textbf{(4)}$$

Armington

$$\mathbf{Q_{sj}} = \mathbf{Q_{\bullet sj}}$$

Krugman

$$\mathbf{Q_{sj}} = \left(\mathbf{N_s Q_{\bullet sj}^{-\rho}} \right)^{-1/\rho} = \mathbf{N_s^{\sigma/(\sigma-1)} Q_{\bullet sj}}$$

Melitz

$$\mathbf{Q_{sj}} = \left[\mathbf{N_{sj} Q_{\bullet sj}^{-\rho}} \right]^{-1/\rho} = \left[\mathbf{N_{sj}} \right]^{\sigma/(\sigma-1)} \mathbf{Q_{\bullet sj}}$$

Profits on the sj link
(getting rid of the k dimension)

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$$\Pi_{ksj} = \mathbf{P}_{ksj}\mathbf{Q}_{ksj} - \left(\frac{\mathbf{W}_s \mathrm{T}_{sj}}{\Phi_{ks}}\right)\mathbf{Q}_{ksj} - \mathbf{F}_{sj}\mathbf{W}_s \qquad \textbf{(5)}$$

Armington

Other equations imply that $\Pi_{ksj} = 0$

Krugman

$$\Pi_{\bullet sj} = \left(\mathbf{P}_{\bullet sj} - \frac{\mathbf{W}_s \mathrm{T}_{sj}}{\Phi_s}\right)\mathbf{Q}_{\bullet sj} = \left(\frac{1}{\sigma - 1}\right)\left(\frac{\mathbf{W}_s \mathrm{T}_{sj}}{\Phi_s}\right)\mathbf{Q}_{\bullet sj}$$

Melitz

$$\Pi_{\bullet sj} = \left(\frac{1}{\sigma - 1}\right)\left(\frac{\mathbf{W}_s \mathrm{T}_{sj}}{\Phi_{ave(s,j)}}\right)\mathbf{Q}_{\bullet sj} - \mathbf{F}_{sj}\mathbf{W}_s \geq 0$$

$$0 = \left(\frac{1}{\sigma - 1}\right)\left(\frac{\mathbf{W}_s \mathrm{T}_{sj}}{\Phi_{min(s,j)}}\right)\mathbf{Q}_{min(s,j)} - \mathbf{F}_{sj}\mathbf{W}_s$$

Additional restrictions: tie down the $\Phi_{min(s,j)}$

Industry profits
(getting rid of the k dimension)

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$$\Pi_s = \sum_j \sum_{k \in S(s,j)} \Pi_{ksj} - N_s H_s W_s \qquad (6)$$

Armington

Other equations imply that $\Pi_s = 0$

Krugman

$$\Pi_s = \sum_j N_s \Pi_{\bullet sj} - N_s H_s W_s = 0$$

Additional restriction: helps tie down N_s

Melitz

$$\Pi_s = \sum_j N_{sj} \Pi_{\bullet sj} - N_s H_s W_s = 0$$

Additional restriction: helps tie down N_s

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Widget employment in country s
(getting rid of the k dimension)

$$L_s = \sum_j \sum_{k \in S(s,j)} \frac{Q_{ksj}}{\Phi_{ksj}} + \sum_j N_{sj} F_{sj} + N_s H_s \qquad (7)$$

Armington

$$L_s = \sum_j \frac{Q_{sj}}{\Phi_s}$$

Krugman

$$L_s = \sum_j \frac{N_s Q_{\bullet sj}}{\Phi_s} + H_s N_s$$

Melitz

$$L_s = \sum_j \frac{N_{sj} Q_{\bullet sj}}{\Phi_{ave(s,j)}} + \sum_j N_{sj} F_{sj} + H_s N_s$$

Loose ends in Melitz: $\Phi_{ave(s,j)}$ and N_{sj}

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$$\Phi_{ave(s,j)} = \beta \Phi_{min(s,j)}, \quad \beta > 1$$

$$\frac{N_{sj}}{N_s} = \Phi_{min(s,j)}^{-\alpha}, \quad \alpha > 0$$

$$Q_{min(s,j)} = \left(\frac{1}{\beta}\right)^{\sigma} Q_{\bullet sj},$$

Melitz relates β and α by:

$$\beta = \left(\frac{\alpha}{\alpha - (\sigma - 1)}\right)^{1/(\sigma-1)} \text{ where } \alpha > (\sigma\text{-}1)$$

Relating β and α

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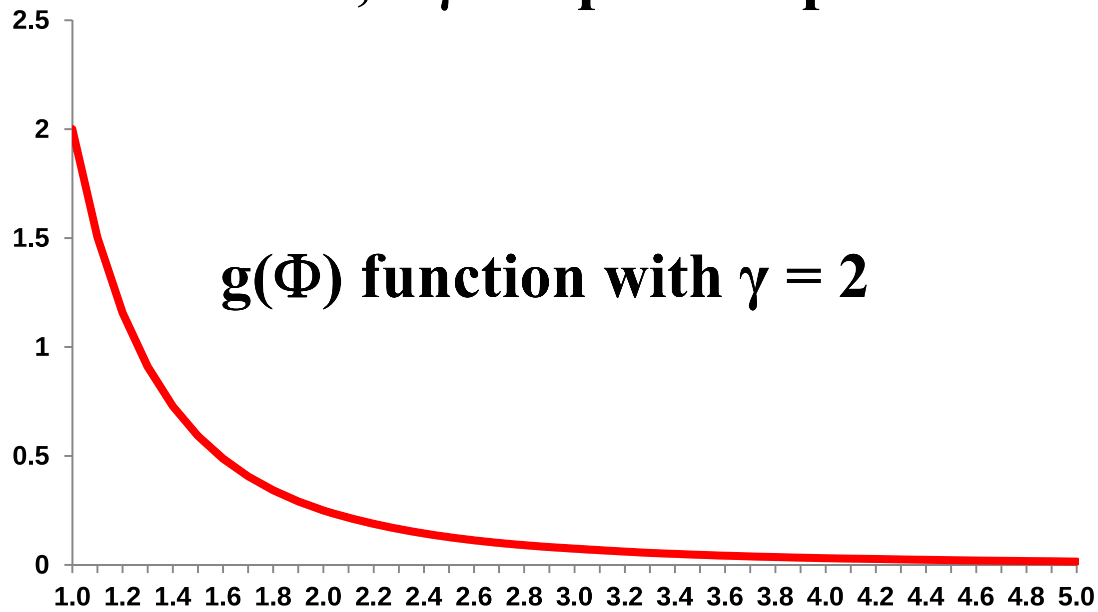
Melitz defines $\Phi_{ave(s,j)}$ as:

$$\Phi_{ave(s,j)} = \left[\int_{\Phi_{min(s,j)}}^{\infty} \Phi^{\sigma-1} \left(\frac{g(\Phi)}{N_{sj}/N_s} \right) d\Phi \right]^{\frac{1}{\sigma-1}}$$

$$\left\{ \text{or} \;\; L_{\bullet sj} = \frac{\sum_{k \in S(s,j)} L_{ksj}}{N_{sj}} \right\}$$

g(Φ), the proportion of firms with productivity Φ, is given by:

$$g(\Phi) = \gamma \Phi^{(-\gamma-1)} \;\; \text{for } 1 < \Phi , \quad \gamma \text{ is a positive parameter}$$



Optimality in the Armington, Krugman and Melitz models

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The planners problem is:

choose Q_{ksd}, $\Phi_{min}(s,d)$, N_s to minimize

$$\sum_s W_s \left[\sum_d \sum_{k \in S(s,d)} N_s g_s(\Phi_k) * \left(\frac{T_{sd} Q_{ksd}}{\Phi_k} + F_{sd} \right) \right] + \sum_s W_s N_s H_s \qquad (3.2)$$

subject to

$$Q_d^{(\sigma-1)/\sigma} = \sum_s \sum_{k \in S(s,d)} N_s g_s(\Phi_k) \delta_{sd} Q_{ksd}^{(\sigma-1)/\sigma} \qquad \forall\, d \qquad (3.3)$$

where

$$S(s,d) = \left\{ \Phi_k : \Phi_k \geq \Phi_{min(s,d)} \right\} \ . \qquad (3.4)$$

An envelope result

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Choose X_1 and X_2

To minimize $\quad \text{Cost} = W_1X_1 + W_2X_2$

Subject to $\quad g(X_1, X_2) = 0$

First order conditions:

$$W_1 = \lambda g_1$$
$$W_2 = \lambda g_2$$
$$g(X_1, X_2) = 0$$

$$\Delta \text{Cost} = \sum_i W_i * \Delta X_i + \sum_i \Delta W_i * X_i$$

But $\sum_i W_i * \Delta X_i = \lambda * \sum_i g_i * \Delta X_i = 0$

Therefore $\Delta \text{Cost} = \sum_i \Delta W_i * X_i$

Conclusion ΔCost **does not depend on changes in X's**

Balistreri-Ruthford decomposition method for solving GE models with Melitz sectors

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Completing the Melitz general equilibrium model

$$R_{csd} = \left(T_{csd} - 1\right)\frac{W_s}{\Phi_{\bullet csd}} N_{csd} Q_{\bullet csd} \qquad (4.1)$$

$$GDP_d = W_d * LTOT_d + \sum_c \sum_s R_{csd} \qquad (4.2)$$

$$LTOT_s = \sum_c L_{cs} \qquad (4.3)$$

$$P_{cd} * Q_{cd} = \mu_{cd} * GDP_d \qquad (4.4)$$

Table 3. The Armington auxiliary model

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	Equation	Dimension	Endogenous variable
(T3.1)	$PA(c,s,d) = \dfrac{WA_s * TA(c,s,d)}{\Phi A(c,s)}$	r^2*n	$PA(c,s,d)$
(T3.2)	$PCA(c,d) = \left(\sum_s \delta A(c,s,d)^{\sigma} * PA(c,s,d)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$	$r*n$	$PCA(c,d)$
(T3.3)	$QA(c,s,d) = QA_{cd} * \left(\delta A(c,s,d) * \dfrac{PCA(c,d)}{PA(c,s,d)} \right)^{\sigma}$	r^2*n	$QA(c,s,d)$
(T3.4)	$LTOTA_s = \sum_{c,d} \left\{ \dfrac{QA(c,s,d)}{\Phi A(c,s)} \right\}$	r	W_s
(T3.5)	$RA(c,s,d) = (TA(c,s,d) - 1) * \left(\dfrac{QA(c,s,d) * WA_s}{\Phi A(c,s)} \right)$	r^2*n	$RA(c,s,d)$
(T3.6)	$GDPA(d) = WA_d * LTOTA_d + \sum_{c,s} RA(c,s,d)$	r	$GDPA(d)$
(T3.7)	$PCA(c,d) * QA_{cd} = \mu_{cd} * GDPA(d)$	$r*n$	Q_{cd}
	Total	$3*r^2n+2*r*n+2*r$	

Connecting the Melitz model and the Armington auxiliary model

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$$\Phi A(c,s) = \frac{\sum_{d} Q_{\bullet csd} N_{csd}}{L_{cs}} \qquad (4.5)$$

$$TA(c,s,d) = 1 + \frac{R_{csd}}{\left(P_{\bullet csd} Q_{\bullet csd} N_{csd} - R_{csd}\right)} \qquad (4.6)$$

$$\delta A(c,s,d) = \left(\frac{\Phi A(c,s) * \dfrac{\left(P_{\bullet csd} Q_{\bullet csd} N_{csd} - R_{csd}\right)}{W_s}}{Q_{cd}} \right)^{\frac{1}{\sigma}} * \left(\frac{\left(\dfrac{W_s * TA(c,s,d)}{\Phi A(c,s)} \right)}{\left(\dfrac{\sum_{s} P_{\bullet csd} Q_{\bullet csd} N_{csd}}{Q_{cd}} \right)} \right) \qquad (4.7)$$

Balistreri-Ruthford decomposition method for solving GE models with Melitz sectors

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Balistreri and Rutherford start by solving each Melitz sector as an independent system of equations based on initial guesses of wage rates and overall demand for sectoral product (W_s and Q_d).

These Melitz computations generate estimates of sectoral productivity and other sectoral variables which are transferred into an Armington multi-sectoral general equilibrium model.

The Armington model is solved to generate estimates of wage rates and overall demand for sectoral product which are fed back into the Melitz sectoral computations.

A full solution of the general equilibrium model with Melitz sectors is obtained when wage rates and overall demand variables emerging from the Armington model coincide with those which were used in the Melitz sectoral computations.

Implication of the Balistreri-Rutherford decomposition

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A solution to a Melitz general equilibrium model can be derived by solving an Armington model with extra shocks to productivity and preference variables.

This suggests that Melitz results can be decomposed into

the primary effect,

the productivity effect, and

the preference effect,

all calculated from an Armington model.

Setting up a numerical model: MelitzGE

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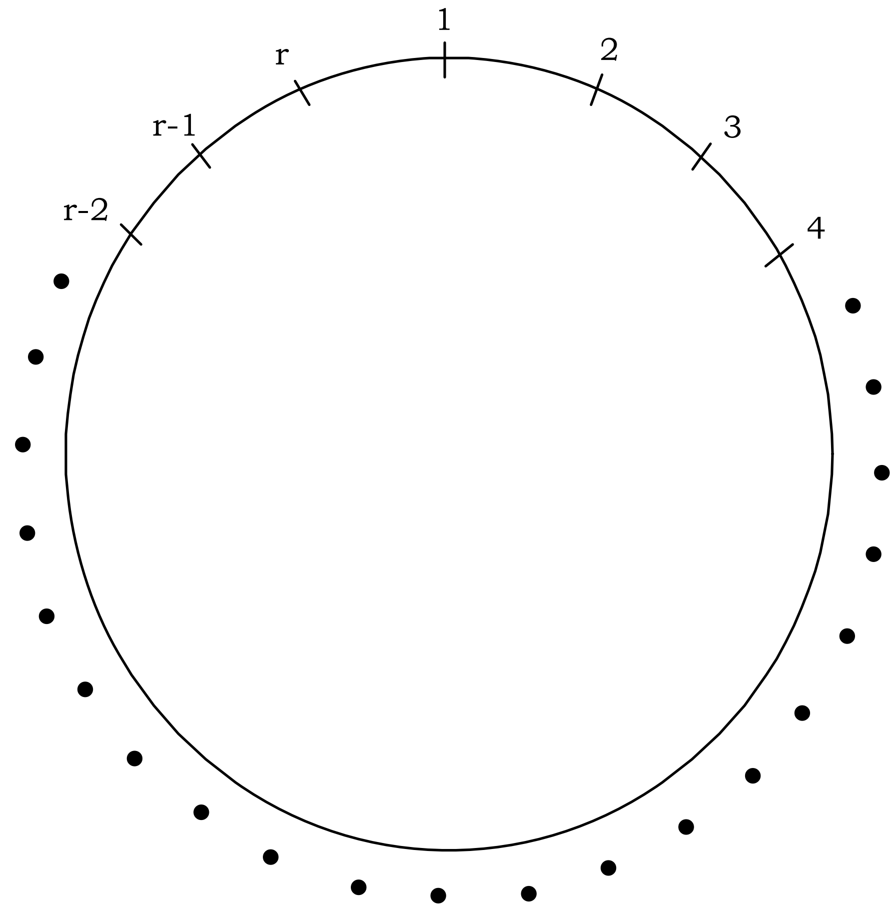

$$\Phi_{\min(c,s,d)} = 1.1 + \frac{(2.0-1.1)}{\mathbf{r}} * 2 * \mathbf{MIN}\left\{|\mathbf{s}-\mathbf{d}|, \mathbf{r}-|\mathbf{s}-\mathbf{d}|\right\} \text{ for all c, s and d} \quad . \qquad (6.1)$$

Test simulations with MelitzGE

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Selected variables	Nominal homogeneity (1)	Scaling fixed costs (2)	Scaling consumption (3)	Increased scale (4)
Exogenous variables				
World average wage rate	1.0	0.0	0.0	0.0
Fixed costs , start up & links				
H_{1s} for all s	0.0	1.0	0.0	0.0
F_{1sd} for all s,d	0.0	1.0	0.0	0.0
Preference variables				
δ_{1s2} for all s	0.0	0.0	0.73588	0.0
Employment by country				
$LTOT_s$ for all s	0.0	0.0	0.0	1.0
Endogenous variables				
Price of composites, P_{11}	1.0	0.35601	0.0	-0.35475
P_{21}	1.0	0.0	0.0	-0.35475
P_{12}	1.0	0.35601	-0.99015	-0.35475
P_{22}	1.0	0.0	0.0	-0.35475
Typical link prices $P_{\bullet csd}$ for all c,s,d	1.0	0.0	0.0	0.0
No. firms on link, N_{1sd} for all s,d	0.0	-0.99015	0.0	1.0
N_{2sd} for all s,d	0.0	0.0	0.0	1.0
Employment by commodity L_{cs} for all c,s	0.0	0.0	0.0	1.0
Consumption by com & country Q_{11}	0.0	-0.35475	0.0	1.35955
Q_{21}	0.0	0.0	0.0	1.35955
Q_{12}	0.0	-0.35475	1.0	1.35955
Q_{22}	0.0	0.0	0.0	1.35955
Trade by typical firm, $Q_{\bullet 1sd}$ for all s,d	0.0	1.0	0.0	0.0
$Q_{\bullet 2sd}$ for all s,d	0.0	0.0	0.0	0.0
Cons. by com, src, & country Q_{1sd} for all s,d	0.0	-0.35475	0.0	1.35955
Q_{2sd} for all s,d	0.0	0.0	0.0	1.35955
Welfare by country welfare(1)	0.0	-0.17753	0.0	1.35955
welfare(2)	0.0	-0.17753	0.49876	1.35955

MelitzGE results for the effects of tariffs imposed by country 2

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Endogenous variables / Shocked exogenous variables	T_{c12}=10 for all c Country d=1	Country d=2	T_{c12}=19 for all c Country d=1	Country d=2	T_{c12}=50 for all c Country d=1	Country d=2
Armington power of tariffs, TA(c,s,d)	0.000	7.180	0.000	13.333	0.000	32.558
Real GDP[1]	-0.006	-0.208	-0.011	-0.643	-0.078	-2.617
Real consumption[1]	-0.824	0.593	-1.436	0.726	-2.908	-0.046
Volume of exports[1]	-18.811	-21.622	-32.008	-36.364	-60.370	-66.389
Volume of imports[1]	-21.622	-18.811	-36.364	-32.008	-66.389	-60.370
Price of exports[1]	1.324	4.958	2.210	9.207	3.536	22.078
Price of imports[1]	4.958	1.324	9.207	2.210	22.078	3.536
Wage rate relative to average world rate[1]	-2.011	2.052	-3.678	3.819	-8.550	9.350
Welfare decomposition						
Welfare(d)	-0.824	0.593	-1.436	0.726	-2.908	-0.046
made up of contributions from changes in::						
Employment	0.000	0.000	0.000	0.000	0.000	0.000
Tax-carrying flows	0.000	-0.164	0.000	-0.497	0.000	-1.994
Terms of trade	-0.818	0.802	-1.425	1.375	-2.832	2.617
Production technology or productivity	-3.332	-2.795	-5.890	-5.021	-12.229	-10.835
Conversion technology or preferences	3.327	2.750	5.879	4.869	12.152	10.165

Is Melitz Armington with a high substitution elasticity?

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	Melitz with σ=3.8		Armington with σ = 8.45	
Shocked exogenous variables	T_{c12}=10 for all c		TA_{c12}=7.18 for all c	
Endogenous variables	Country d=1	Country d=2	Country d=1	Country d=2
Armington power of tariffs, TA(c,s,d)	0.000	7.180	0.000	7.180
Real consumption[1]	-0.824	0.593	-0.830	0.655
Volume of exports[1]	-18.811	-21.622	-18.789	-21.682
Volume of imports[1]	-21.622	-18.811	-21.682	-18.789
Welfare decomposition				
Welfare(d)	-0.824	0.593	-0.830	0.655
made up of contributions from changes in::				
Employment	0.000	0.000	0.000	0.000
Tax-carrying flows	0.000	-0.164	0.000	-0.161
Terms of trade	-0.818	0.802	-0.830	0.816
Production technology or productivity	-3.332	-2.795	0.0	0.0
Conversion technology or preferences	3.327	2.750	0.0	0.0

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A full solution of the general equilibrium model with Melitz sectors is obtained when wage rates and overall demand variables emerging from the Armington model coincide with those which were used in the Melitz sectoral computations.

Concluding remarks: trade in CGE modelling

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Pre 1970s Heckscher-Ohlin - imports and domestic products identical, constant returns to scale, perfect competition
→ big gains from trade but unrealistic specialization

1970s- Armington - import/domestic imperfect substitution, constant returns to scale, perfect competition
→ big negative terms-of-trade effects from cutting tariffs even for small countries, often dominate positive efficiency gain

1980s- Krugman - variety at firm level, not country level, increasing returns to scale, monopolistic competition among identical firms
→ still get big negative terms-of-trade effects but potential extra welfare from additional variety and increasing returns to scale

2003- Melitz - variety at firm level, increasing returns to scale, monopolistic competition among firms with different productivity
→ still get big negative terms-of-trade effects but potential extra welfare from additional variety, increasing returns to scale, and pro-trade productivity effect

Concluding remarks

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We have shown that

Armington is a special case of Krugman

Krugman is a special case of Melitz, and

Melitz is a special cases of a more general model

Despite increasing returns to scale, imperfect competition, separate variety for each firm, and different productivity levels across firms, the Melitz model produces an optimal market outcome.

-- envelope theorems work

Melitz solutions can be calculated in an Armington model with extra shocks to productivity and preferences.

Concluding remarks

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Melitz welfare results can be decomposed into
primary effect
productivity effect
preference effect
all calculated in an Armington model.

Productivity and preference effects offset - envelope theorem

Melitz results can be reproduced in an Armington model with a high Armington elasticity