An introduction to Computable General Equilibrium (CGE)

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Overview

- Why 'General Equilibrium'?
- What 'Computable'?
- Comparative statics
- Uses and pitfalls
- Example of a CGE model: ORANI-G

References

- Piermartini and Teh (2005)
- Dixon (2006)
- Horridge (2003)
- McDougall (1999)

Why ‘General Equilibrium’?

Linkages between different sectors (upstream/downstream; domestic/foreign, infrastructure/final production), agents (consumers, producers, governments). economies (developing/developed), generations (current and future resource availability/technologies/policies).

Optimisation and adding up issues

Linkages between different sectors

- Resources sector boom may lead to decline in manufacturing sector ("Dutch Disease").
- Domestic CO_2 emissions restrictions on electricity generation sector may lead to rising electricity prices and affecting export performance.
- Climate change may impact on agriculture which in turns impacts on trade; conversely, trade policies can help, or hinder, the process of adaptation in agriculture to climate change.

Linkages between different economies

- Chinese economic growth can lead to an expansion in the Australian resources sector.
- EU climate change policies (reducing CO_2 emissions) can lead to a decline of coal export from South Africa to the EU, but this can also lead to an increase in manufacturing exports from South Africa, this depends on the interactions between EU – Chinese – South African economies in response to EU climate policies.

Linkages between generations

- Current GHG emissions add to future GHG concentrations and climate change.
- Current investments induce future technological change.
- Current climate change adaptation/mitigation can help on reducing future costs.

Optimisation, adding up issues

- Producers and consumers are ‘rational’ decision makers subject to resource and technological constraints; governments are subject to budgetary constraints; countries are subject to trade and investment constraints.
- Income is the opposite side of expenditure, imports of exports, and costs of prices.

  A ‘general equilibrium’ framework is necessary to ensure consistency and adding up constraint for all actions - taking into account ‘leakages’ and/or ‘externalities’.

What 'Computable'?

Policy targets

Economic/environmental impacts

Behavioural / institutional responses

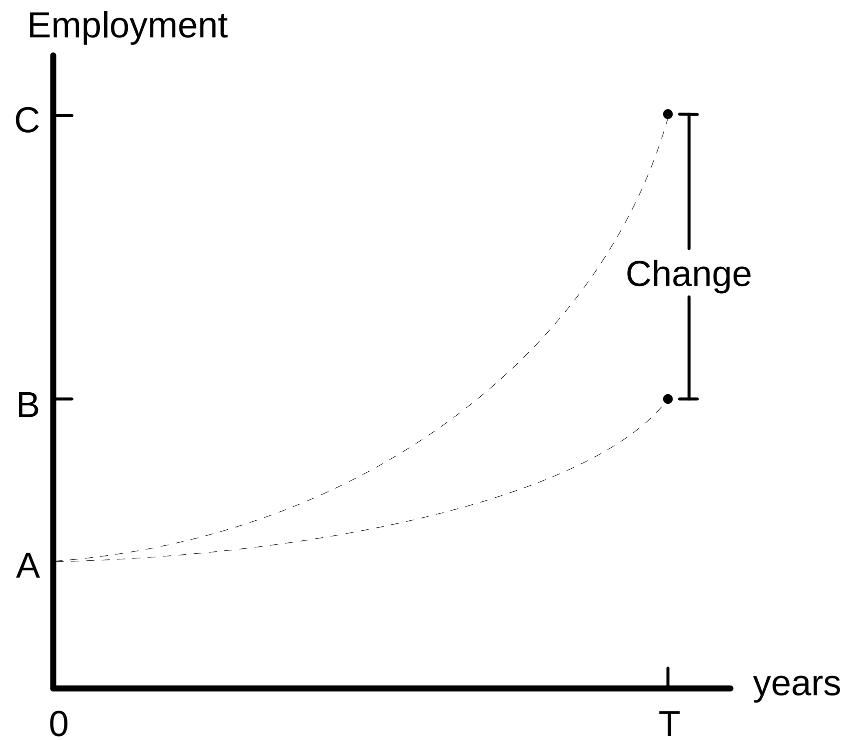
Targets, impacts and responses

- Trade liberalisation needs to have targets in terms of tariff reductions; climate change policies need to have targets in terms of GHG emissions reductions.
- Economic costs of policies, impacts on the economic and natural environment need to be quantified.
- Responses from individual economic agents/sectors/regions need to be assessable and measurable.

Comparative statics

- CGE models answer the "what if" questions tied to a particular base year.
- Base year data is to be 'balanced' (i.e. issues like inventory, entrepot trade, cyclical or temporary effects of climate and weather on agricultural productions need to be sorted out)
- Results from a static CGE model show what is the shift in this balanced position if certain 'exogenous' variables are shocked or disturbed by a certain percentage.
- Static CGE model is atemporal, it tells us nothing of the adjustment paths (the dotted lines shown in the following Figure 1)

Comparative statics



Comparative-static interpretation of results
Source: Horridge, M. "ORANI-G: A Generic Single-Country Computable General Equilibrium Model" Notes prepared for the Practical GE Modelling Course June 23-27, 2003. Centre of Policy Studies and Impact Project, Monash University, Australia.
http://www.monash.edu.au/policy/ftp/oranig/oranig03.zip

Policy Analysis versus Forecasting

- The economy at time ‘0’ (base year) is not the same as the economy in the forecast year ‘T’.
- The MONASH Model - Unlike ORANI, has a strong forecasting capability due to:
  – a more detailed specification of intertemporal (ie dynamic) relationships;
  – greater use of up-to-date data; and
  – enhancements which allow the model to take on information from specialist forecasting organisations and from recent historic trends

Monash - four modes of analysis

- *Historical,* where changes in technology, consumer preferences, positions of foreign demand curves for domestic products and numerous other naturally exogenous trade variables are estimated;
- *Decomposition,* where we explain periods of economic history in terms of driving factors such as changes in technology, consumer preferences and trade variables;
- *Forecast,* where we derive basecase forecasts for industries, occupations and regions that are consistent with trends from historical simulations and with available expert opinions;
- *Policy,* where we derive deviations from basecase forecast paths caused by assumed policies.

Uses and Pitfalls

Uses

Pitfalls

Validation

Uses

- CGE models are like a computer ‘platforms’ on which certain designs and applications can be integrated and applied to throw light on certain problems/issues.
- Results from CGE models are like computer-based simulations or laboratory experiments, they can be used to assist policy makers in deliberating and ‘thinking’ about all the complex and sometimes unexpected interactions in a national/ global economy.

Uses

- Use CGE models in the analysis of issues which involve a lot of *interactions* between players, sectors, instruments, policies.
  - e.g. Trade policies, with gainers and losers, taxes and subsidies, movements of capital and labour across sectors and regions.
- Use CGE models in the analysis of problems that require *adding up* in the results
  - e.g. Tax reform policies with revenues/expenditures need adding up.
  - e.g. Energy/climate change policies which involve substitution and harmonisation as well as 'leakages' and 'externalities'

Pitfalls

- Numbers can give a false sense of accuracy or reality - 'garbage in garbage out', therefore, 'data in' need to be checked for accuracy and consistency, 'results out' need to be tested for robustness - i.e. insensitivity with respect to assumptions and/or parameters used.

Pitfalls

- Do not use CGE models for problems which are either too simple or ‘partial’ i.e. not involving a great deal of interactions/feedback or indirect effects; a partial analysis in this case can be more cost effective.

Validation

- Ex-post validation of CGE model results can be difficult because often results are not simple forecasts but comparative statics analysis (CSA).
- To ex-post validate a CSA set of ('timeless') results we need to compare results with actual (ex-post) data but (ex-post) data need to be purged of all intervening exogenous events
- Alternatively, to compare actual (ex-post) data with ('dated') simulations, results need to be re-run with exogenous variables re-set to values which are close to their actual values.

e.g. ORANI-G Model

Data $\leftrightarrow$ Structure $\leftrightarrow$ Equations
Current production (CES, CET, Leontief)
Investment (Leontief)
Demand (CES, Klein-Rubin)
Zero pure profit (economic accounting)
Imports, exports, taxes, transfer (National Income Accounting)

ORANI-G

- ORANI was originally a CGE model of Australia, it has been extended in several ways and the current version ORANI-G has been used as a ‘template’ for the creation of many other CGE models: for South Africa, Vietnam, Indonesia, South Korea, Thailand, the Philippines, Pakistan, Denmark, both Chinas, and Fiji.

ORANI-G training courses

- The Centre of Policy Studies
  and the IMPACT Project
  Monash University
- June 2010 Practical CGE modeling course
  venue: "Berkley's on Ann", Rendezvous Hotel,
  Brisbane, Australia
- Monday June 28 to Friday July 2

  http://www.monash.edu.au/policy/courses.htm

ORANI Model – data base

		Domestic Industries (Current production)	Absorption or Final Demands: Investment	Household Consumption	Government Consumption	Exports	Change in Inventories
Basic flows of Domestic commodities / Basic flows of Imported commodities		V1BAS $C \times S \times I$	V2BAS $C \times S \times I$	V3BAS $C \times S \times 1$	V4BAS $C \times S \times 1$	V5BAS $C \times S \times 1$	V6BAS $C \times S \times 1$
Margin type *m* on	domestic flows / imports flows	V1MAR $M \times C \times S \times I$	V2MAR $M \times C \times S \times I$	V3MAR $M \times C \times S \times 1$	V4MAR $M \times C \times S \times 1$	V5MAR $M \times C \times S \times 1$	
Taxes on	domestic flows / imports flows	V1TAX $C \times S \times I$	V2TAX $C \times S \times 1$	V3TAX $C \times S \times 1$	V4TAX $C \times S \times 1$	V5TAX $C \times S \times 1$	
Primary Factors	Labour	V1LAB $O \times I$					
	Capital	V1CAP $1 \times I$					
	Land	V1LND $1 \times I$					
Production tax		V1PTX $1 \times I$					
Other costs		V1OCT $1 \times I$					
Domestic commodities		MAKE $C \times I$	V0TAR $C \times 1$				

USE matrix

Joint production (or Make) matrix

Import tariff

C commodities

S sources (domestic, imported)

I industries

M margins

O occupations

MAKE matrix for single-product industries

I industries

C commodities

	1	2	3	4	5
1	12				
2		15			
3			20		
4				50	
5					37

MAKE matrix for *muliple*-product industries

h industries

g commodities

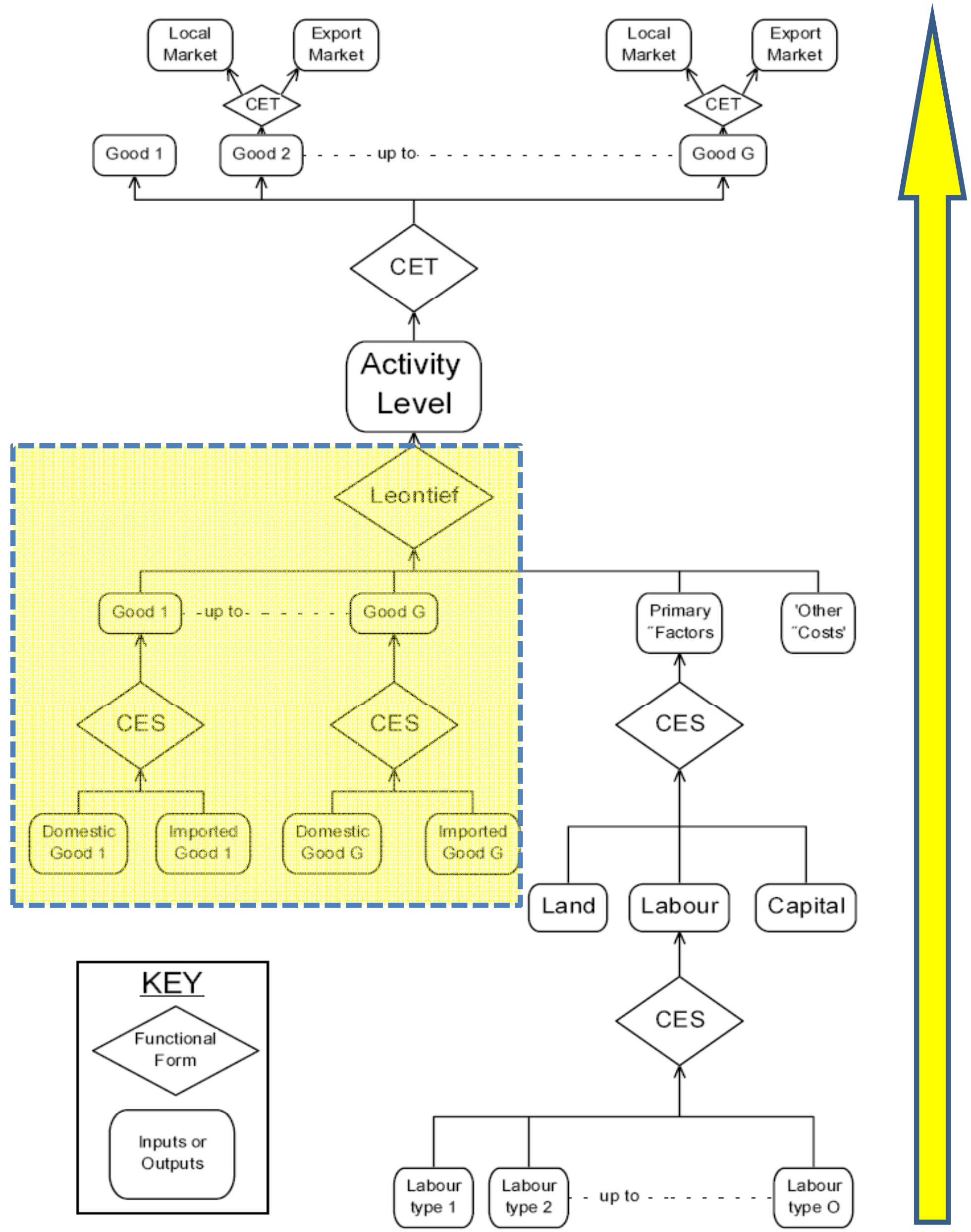
	1	2	3	4	5
1	5	6			
2	7	9			
3			20		
4				50	
5					37

ORANI-G

- Structure of Production
  - Working ‘upwards’: primary factors, intermediate goods (including margins, commodity or user-specific indirect taxes), ad valorem production tax, other costs → output
  - For single-output industry: production output ≡ commodity demanded.
  - For multi-product industry: activity level ‘transformed’ into outputs of various commodities (e.g. via a CET transformation function).

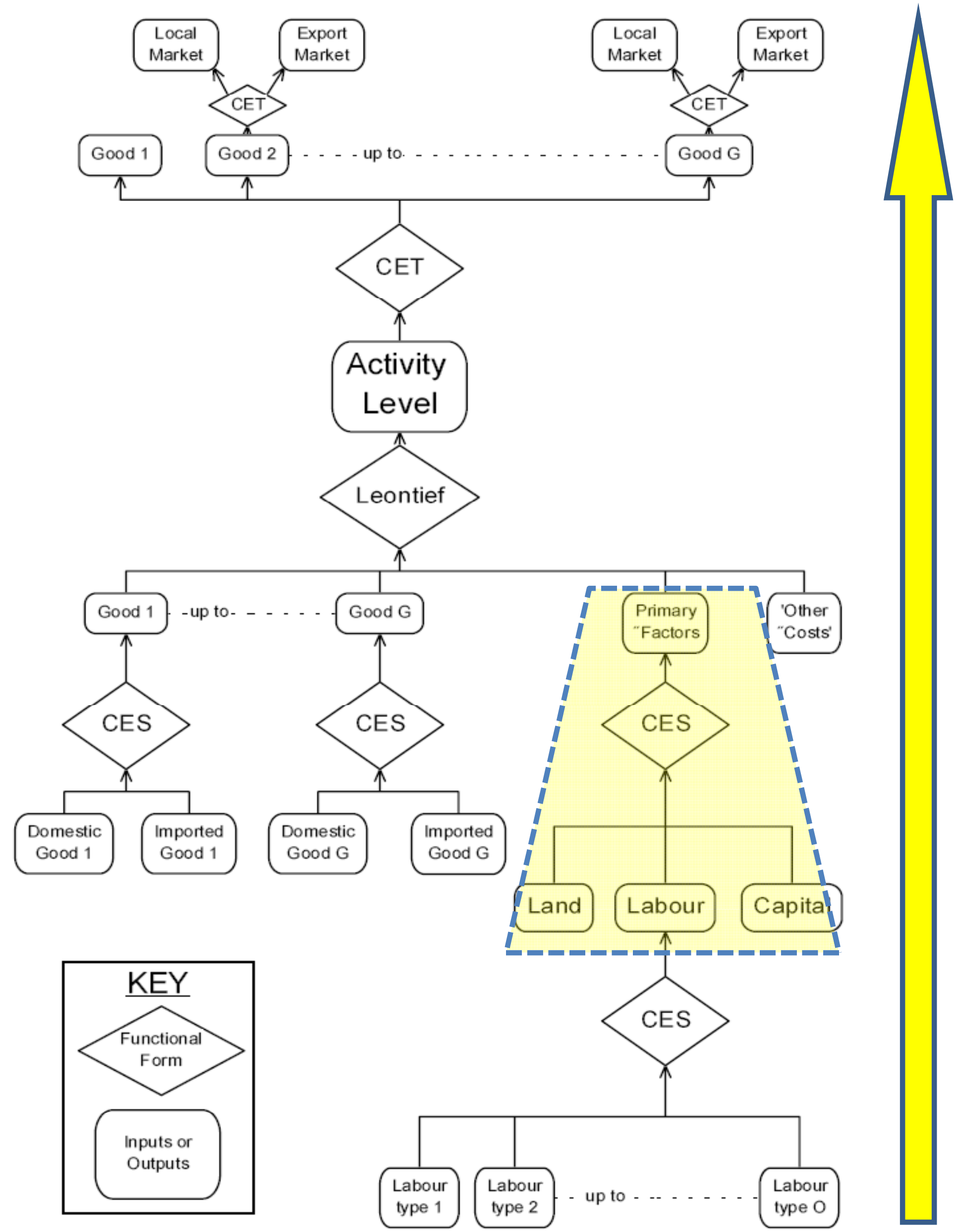
ORANI Model – data base and current production structure

		Domestic Industries (Current production)
Basic flows of Domestic commodities		V1BAS *C* x *S* x *I*
Basic flows of Imported commodities		
Margin type *m* on	domestic flows	V1MAR *M* x *C* x *S* x *I*
	imports flows	
Taxes on	domestic flows	V1TAX *C* x *S* x *I*
	imports flows	
Primary Factors	Labour	V1LAB *O* x *I*
	Capital	V1CAP 1 x *I*
	Land	V1LND 1 x *I*
Production tax		V1PTX 1 x *I*
Other costs		V1OCT 1 x *I*
Domestic commodities		MAKE *C* x *I*



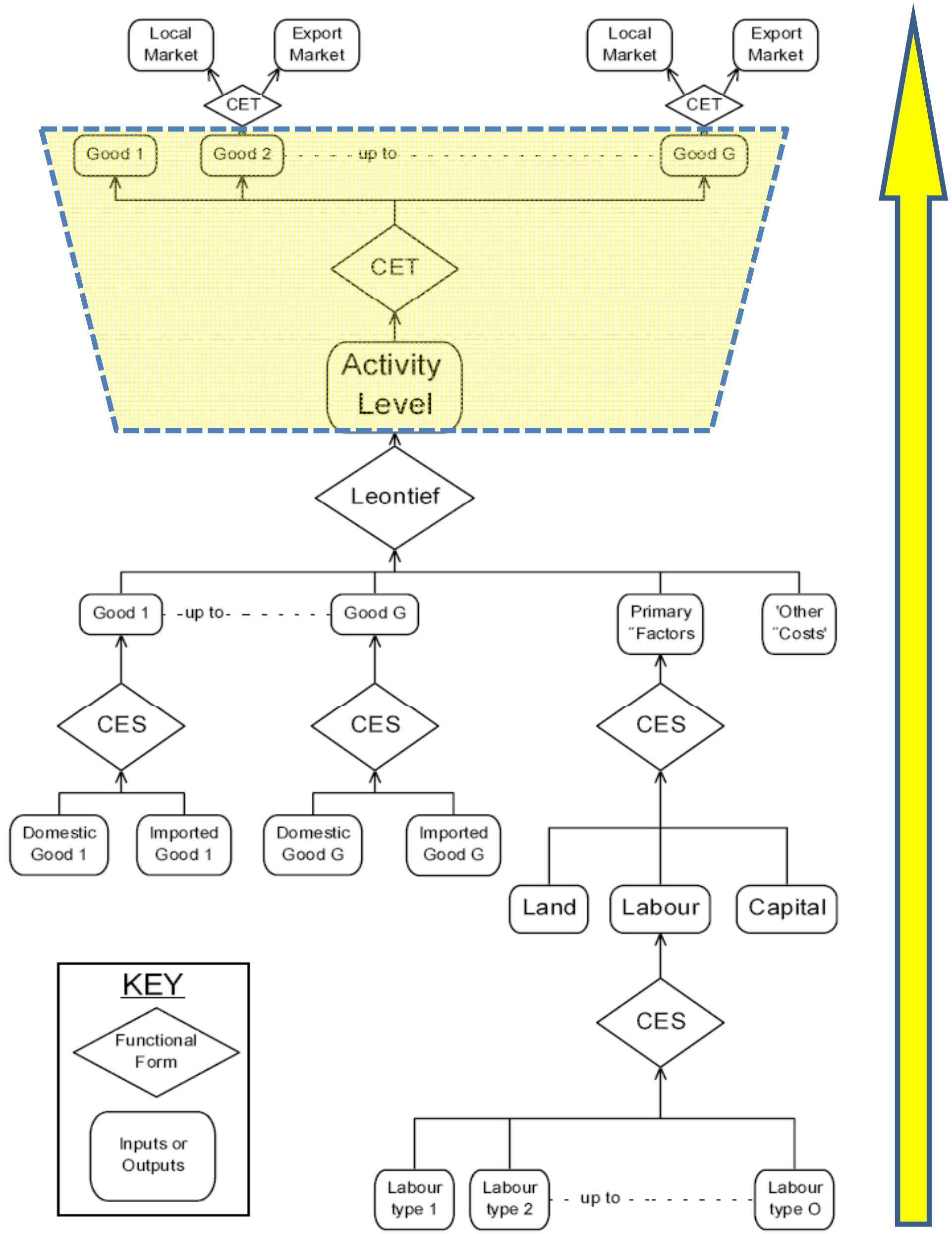
ORANI Model – data base and current production structure

		Domestic Industries (Current production)
Basic flows of Domestic commodities		V1BAS $C \times S \times I$
Basic flows of Imported commodities		
Margin type *m* on	domestic flows	V1MAR $M \times C \times S \times I$
	imports flows	
Taxes on	domestic flows	V1TAX $C \times S \times I$
	imports flows	
Primary Factors	Labour	V1LAB $O \times I$
	Capital	V1CAP $1 \times I$
	Land	V1LND $1 \times I$
Production tax		V1PTX $1 \times I$
Other costs		V1OCT $1 \times I$
Domestic commodities		MAKE $C \times I$



ORANI Model – data base and current production structure

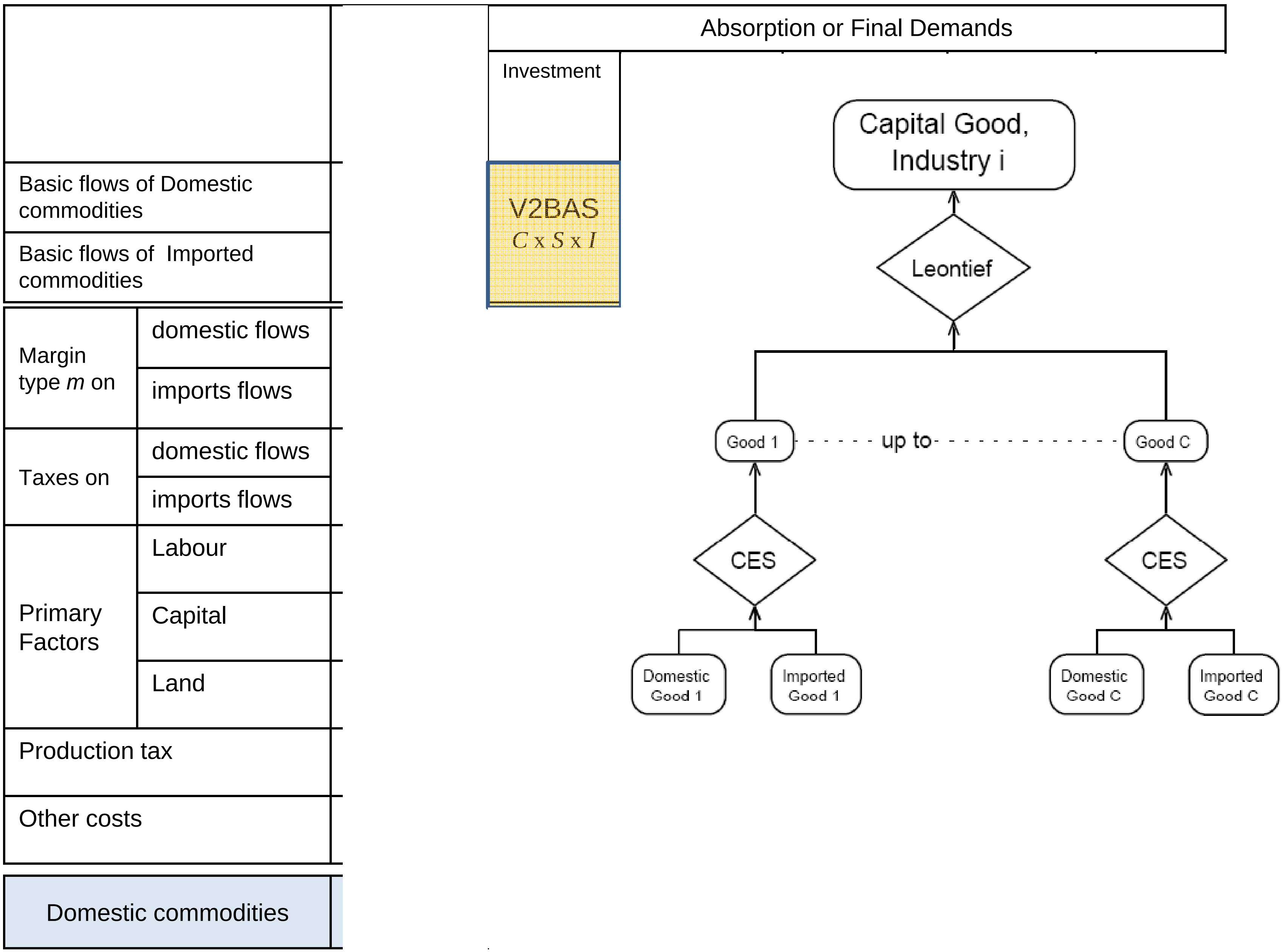
		Domestic Industries (Current production)
Basic flows of Domestic commodities		V1BAS C x S x I
Basic flows of Imported commodities		
Margin type m on	domestic flows	V1MAR M x C x S x I
	imports flows	
Taxes on	domestic flows	V1TAX C x S x I
	imports flows	
Primary Factors	Labour	V1LAB O x I
	Capital	V1CAP 1 x I
	Land	V1LND 1 x I
Production tax		V1PTX 1 x I
Other costs		V1OCT 1 x I
Domestic commodities		MAKE C x I



ORANI-G

- Structure of Demand: (a) Investment goods
  - Capital is assumed to be produced with inputs of domestically produced and imported commodities. The production function has the same nested structure as that which governs intermediate inputs to current production, but with no primary factors used directly as inputs to capital formation

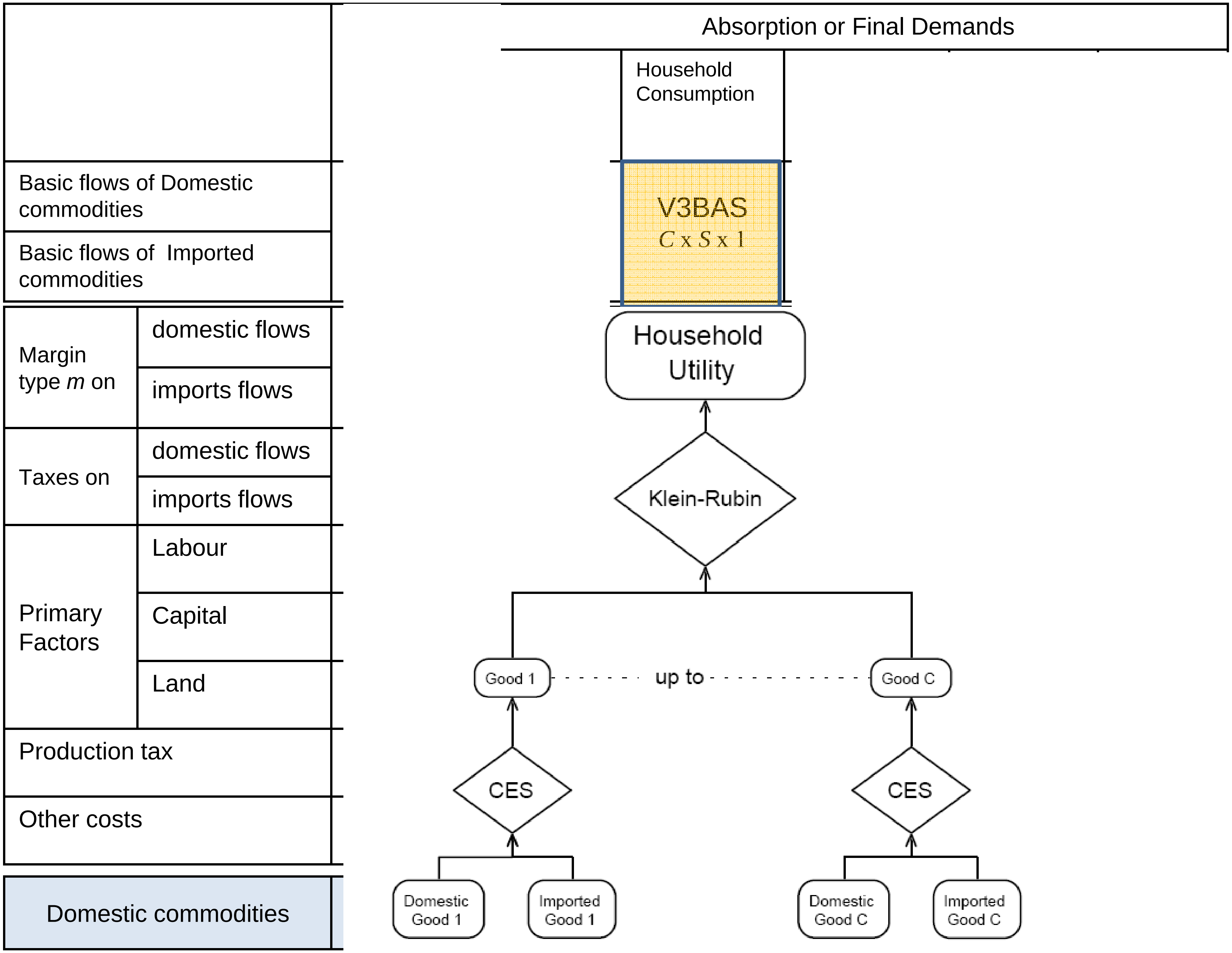
ORANI Model – data base



ORANI-G

- Structure of Demand: (b) Consumption goods
  - The nesting structure for household demand is nearly identical to that for investment demand. The only difference is that commodity composites are aggregated by a Klein-Rubin, rather than a Leontief, function, leading to the linear expenditure system (LES).

ORANI Model – data base



Basic economic theories used in CGE Modelling

- Producer: Cost minimisation → Demand for factors of production, cost function.
- Consumer: Utility Maximisation → Demand for commodities, Indirect Utility, Expenditure function
- Important parameters: elasticities of demand/supply/substitution.
- Functional forms: Cobb-Douglas (C-D), Constant Elasticity of Substitution (CES), Constant Elasticity of Transformation (CET), others.

CES – basic building block

Minimize $C = P_1X_1 + P_2X_2$

s.t. $Y = \alpha\,[\delta_1X_1^{\rho} + \delta_2X_2^{\rho}]^{1/\rho}$

or:

Maximize $U = \alpha\,[\delta_1X_1^{\rho} + \delta_2X_2^{\rho}]^{1/\rho}$

s.t. $M = P_1X_1 + P_2X_2$

CES – producer cost minimisation

$\mathcal{L} = P_1X_1 + P_2X_2 + \Lambda\{Y - \alpha[\delta_1X_1^{\rho} + \delta_2X_2^{\rho}]^{1/\rho}\}$

$\partial\mathcal{L}/\partial X_1 = 0 \Rightarrow P_1 = \Lambda\alpha\delta_1X_1^{\rho-1}[\delta_1X_1^{\rho} + \delta_2X_2^{\rho}]^{(1/\rho)-1}$

$\partial\mathcal{L}/\partial X_2 = 0 \Rightarrow P_2 = \Lambda\alpha\delta_2X_2^{\rho-1}[\delta_1X_1^{\rho} + \delta_2X_2^{\rho}]^{(1/\rho)-1}$

$C = P_1X_1 + P_2X_2 = \Lambda Y$ or $\Lambda = C/Y$ (unit cost)

Noting that $(Y/\alpha)^{\rho} = [\delta_1X_1^{\rho} + \delta_2X_2^{\rho}]$, solve for X_1 and X_2:

$P_1 = \Lambda\alpha\delta_1X_1^{\rho-1}[Y/\alpha]^{(1-\rho)}$ → $X_1 = (Y/\alpha)[\Lambda\alpha\delta_1/P_1]^{1/(1-\rho)}$

$P_2 = \Lambda\alpha\delta_2X_2^{\rho-1}[Y/\alpha]^{(1-\rho)}$ → $X_2 = (Y/\alpha)[\Lambda\alpha\delta_2/P_2]^{1/(1-\rho)}$

Cost function (substituting X_1 and X_2 into C):

$C = (Y/\alpha)[\delta_1^{\sigma}P_1^{1-\sigma} + \delta_2^{\sigma)}P_2^{1-\sigma}]^{1/(1-\sigma)}$ where $\sigma = 1/(1-\rho)$

→ P (unit cost or price) = $(1/\alpha)[\delta_1^{\sigma}P_1^{1-\sigma} + \delta_2^{\sigma}P_2^{1-\sigma}]^{1/(1-\sigma)}$

CES – producer cost minimisation

Cost shares: ($S_1 = X_1P_1/C$; $S_2 = X_2P_2/C$)

$$S_1 = \delta_1 X_1^{\rho} / [\delta X_1^{\rho} + \delta_2 X_2^{\rho}];\text{ or}$$

$$S_1 = \delta_1 P_1^{1-\sigma} / [\delta_1 P_1^{1-\sigma} + \delta_2 P_2^{1-\sigma}];$$

Similarly for S_2 :

$$S_2 = \delta_2 X_2^{\rho} / [\delta_1 X_1^{\rho} + \delta_2 X_2^{\rho}];\text{ or}$$

$$S_2 = \delta_2 P_2^{1-\sigma} / [\delta_1 P_1^{1-\sigma} + \delta_2 P_2^{1-\sigma}];$$

CES – consumer utility maximisation

$$\mathcal{L} = \alpha[\delta_1 X_1^{\rho} + \delta_2 X_2^{\rho}]^{(1/\rho)} + \Lambda\{M - P_1X_1 - P_2X_2\}$$

$$\partial\mathcal{L}/\partial X_1 = 0 \Rightarrow \Lambda P_1 = \alpha\delta_1 X_1^{\rho-1}[\delta_1 X_1^{\rho} + \delta_2 X_2^{\rho}]^{(1/\rho)-1}$$

$$\partial\mathcal{L}/\partial X_2 = 0 \Rightarrow \Lambda P_2 = \alpha\delta_2 X_2^{\rho-1}[\delta_1 X_1^{\rho} + \delta_2 X_2^{\rho}]^{(1/\rho)-1}$$

$U = \Lambda[P_1X_1 + P_2X_2]$ or $\Lambda = U/M$ (utility of money)

Noting that $(U/\alpha)^{\rho} = [\delta_1 X_1^{\rho} + \delta_2 X_2^{\rho}]$, solve for X_1 and X_2:

$$\Lambda P_1 = \alpha\delta_1 X_1^{\rho-1}[U/\alpha]^{(1-\rho)} \rightarrow X_1 = (U/\alpha)[\alpha\delta_1/(\Lambda P_1)]^{1/(1-\rho)}$$

$$\Lambda P_2 = \alpha\delta_2 X_2^{\rho-1}[U/\alpha]^{(1-\rho)} \rightarrow X_2 = (U/\alpha)[\alpha\delta_2/(\Lambda P_2)]^{1/(1-\rho)}$$

Indirect utility function (substituting X_1 and X_2 into U):

$$V = \alpha M\,[\delta_1^{\sigma} P_1^{1-\sigma} + \delta_2^{\sigma} P_2^{1-\sigma}]^{-1/(1-\sigma)}$$ where $\sigma = 1/(1-\rho)$

Expenditure function (inverting V):

$$M = (U/\alpha)\,[\delta_1^{\sigma} P_1^{1-\sigma} + \delta_2^{\sigma} P_2^{1-\sigma}]^{1/(1-\sigma)}$$

CES – consumer utility maximisation

Cost shares: ($S_1 = X_1P_1/M$; $S_2 = X_2P_2/M$)

$$S_1 = \delta X_1^{\rho} / [\delta X_1^{\rho} + \delta_2 X_2^{\rho}]; \text{ or}$$

$$S_1 = \delta P_1^{1-\sigma} / [\delta P_1^{1-\sigma} + \delta_2 P_2^{1-\sigma}];$$

Similarly for S_2 :

$$S_2 = \delta_2 X_2^{\rho} / [\delta X_1^{\rho} + \delta_2 X_2^{\rho}]; \text{ or}$$

$$S_2 = \delta_2 P_2^{1-\sigma} / [\delta P_1^{1-\sigma} + \delta_2 P_2^{1-\sigma}];$$

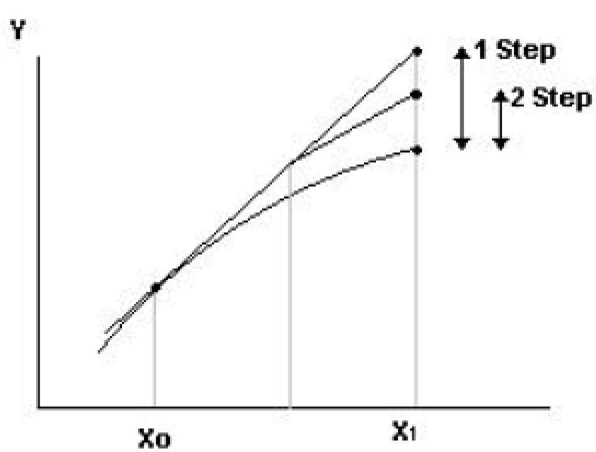
Linearisation

- <u>Definition</u>: if *Z* is a variable, let *dZ* be the *absolute change* in *Z* (we use the symbol '*d*' if it is a very small, or differential change, and *D*;if it is a relatively large or finite change); (*dZ*/*Z*) is the *relative* or (if multiplied by 100) *percentage change* in *Z*. We note that (*dZ*/*Z*) is also equal to *d*ln*Z* (i.e. the differential of the logarithmic function of *Z*). Logarithmic is a useful function because it converts a product into summation, which is easier to handle.

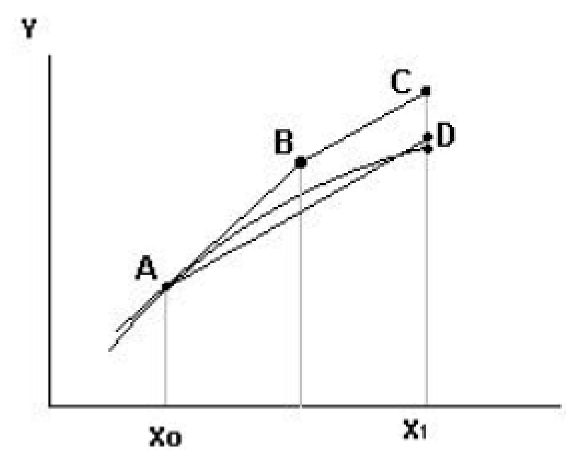
Linearisation is only a numerical method…

- for solving a non-linear system of equations,
- capable of achieving an ‘exact’ solution,
- depending on the number of steps in approaching a ‘solution’ and a particular method used:

Solution Using Euler’s Method



Solution using Gragg’s Method



Linearisation – some useful formulae

$Z = X.Y$

$dZ = XdY + Y\,dX$

$dZ/Z = dY/Y + dX/X$

$d\ln Z = d\ln X + d\ln Y$

$(\%Z) = (\%X) + (\%Y)$

$z = x + y$

$Z = X + Y$

$dZ = dX + dY$

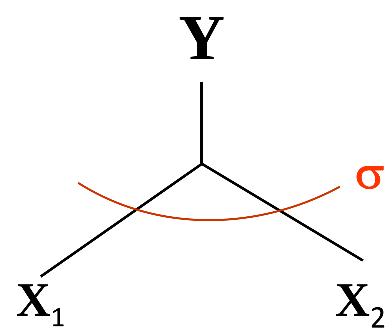
$dZ/Z = (X/Z)dX/X + (Y/Z)dY/Y$

$d\ln Z = S_X d\ln X + S_Y d\ln Y$

$(\%Z) = S_X\,(\%X) + S_Y\,(\%Y)$

$z = S_X x + S_Y y$

Linearisation – CES production function



$Y = \alpha[\delta_1 X_1^{\rho} + \delta_2 X_2^{\rho}]^{1/\rho}$

$d\ln Y = (1/\rho)\, d\ln [\delta_1 X_1^{\rho} + \delta_2 X_2^{\rho}]$

$d\ln [\delta_1 X_1^{\rho} + \delta_2 X_2^{\rho}] = S_A\, d\ln A + S_B\, d\ln B$

where $A = \delta_1 X_1^{\rho}$; $B = \delta_2 X_2^{\rho}$

$S_A = \delta_1 X_1^{\rho} / [\delta_1 X_1^{\rho} + \delta_2 X_2^{\rho}] = S_1 = P_1 X_1 / [P_1 X_1 + P_2 X_2]$;

$d\ln A = \rho\, d\ln X_1$; similarly for S_B and $d\ln B$.

$d\ln Y = S_1\, d\ln X_1 + S_2\, d\ln X_2$

$y = S_1 x_1 + S_2 x_2$

Linearisation – CES factor demand

Recall:

$P_1 = \Lambda\alpha\delta_1 X_1^{\rho-1}[Y/\alpha]^{(1-\rho)}$ → $X_1 = (Y/\alpha)[\Lambda\alpha\delta_1/P_1]^{1/(1-\rho)}$

$P_2 = \Lambda\alpha\delta_2 X_2^{\rho-1}[Y/\alpha]^{(1-\rho)}$ → $X_2 = (Y/\alpha)[\Lambda\alpha\delta_2/P_2]^{1/(1-\rho)}$

Recall also: $C = P_1X_1 + P_2X_2 = \Lambda Y$ or $\Lambda = C/Y$ (unit cost)

In percentage change terms:

$x_1 = y - \sigma[p_1 - \lambda]$; where $\sigma = 1/(1-\rho)$;

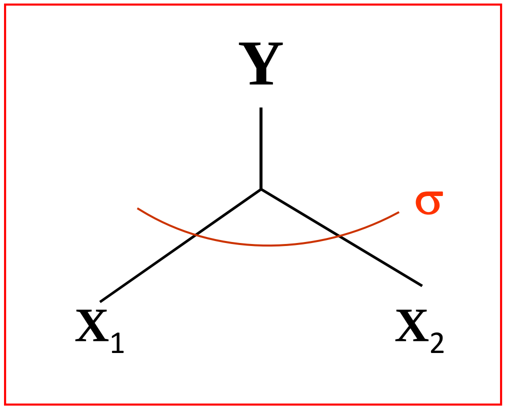
λ = % change in unit cost Λ; $\lambda = c - y$;

$c = S_1(p_1 + x_1) + S_2(p_2 + x_2)$;

Recall previously: $y = S_1x_1 + S_2x_2$

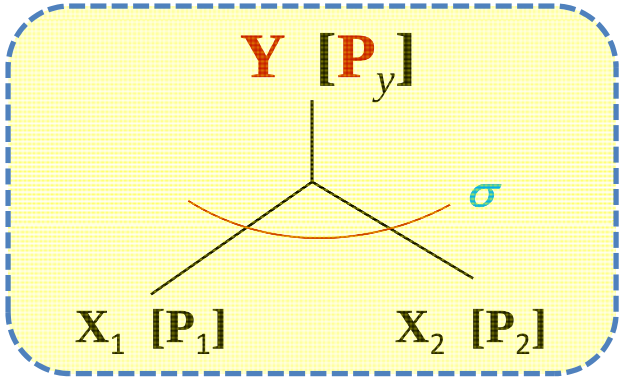
$\therefore\ \lambda = S_1p_1 + S_2p_2 = p_{ave}$

$\therefore\ x_1 = y - \sigma[p_1 - p_{ave}]$ similarly for x_2



CES – summary – closure issue

Absolute level: (UPPER case):



Cost share: $S_1 = P_1X_1/(P_1X_1 + P_2X_2)$

$S_2 = P_2X_2/(P_1X_1 + P_2X_2)$

Total quantity: Y

Unit price: P_y

Total cost: $P_yY = (P_1X_1 + P_2X_2)$

Percentage change: (lower case, **bold**)

'output' from 'inputs':

% change in quantity: $\boldsymbol{y} = S_1\boldsymbol{x}_1 + S_2\boldsymbol{x}_2$ EQUATION 1

% change in price: $\boldsymbol{y} = S_1\boldsymbol{x}_1 + S_2\boldsymbol{x}_2$ EQUATION 2

'inputs' from 'output':

% change in input 1: $\boldsymbol{x}_1 = \boldsymbol{y} - \sigma[\boldsymbol{p}_1 - \boldsymbol{p}_y]$ EQUATION 1

% change in input 2: $\boldsymbol{x}_2 = \boldsymbol{y} - \sigma[\boldsymbol{p}_2 - \boldsymbol{p}_y]$ EQUATION 2

6 variables, 2 equations:→ 2 variables are 'endogenous', 4 variables must be EXOG

e.g. Closure 1: EXOG: $\boldsymbol{p}_y, \boldsymbol{y}, \boldsymbol{p}_1, \boldsymbol{p}_2$ → ENDOG: $\boldsymbol{x}_1, \boldsymbol{x}_2$

e.g. Closure 2: EXOG: $\boldsymbol{p}_y, \boldsymbol{y}, \boldsymbol{x}_1, \boldsymbol{x}_2$ → ENDOG: $\boldsymbol{p}_1, \boldsymbol{p}_2$

e.g. Closure 3: EXOG: $\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{x}_1, \boldsymbol{x}_2$ → ENDOG: $\boldsymbol{p}_y, \boldsymbol{y}$

CES – calibration

$$Y = \alpha\, [\delta_1 X_1^{\rho} + \delta_2 X_2^{\rho}]^{1/\rho}$$

Using the 'calibrated' form:

$$(Y/ Y_{0)} = [\delta_1 (X_1/X^0{}_1)^{\rho} + \delta_2 (X_2/X^0{}_2)^{\rho}]^{1/\rho}$$

where the superscript '0' denotes base year value,

then, the only parameter to be calibrated is the distribution parameter δ.

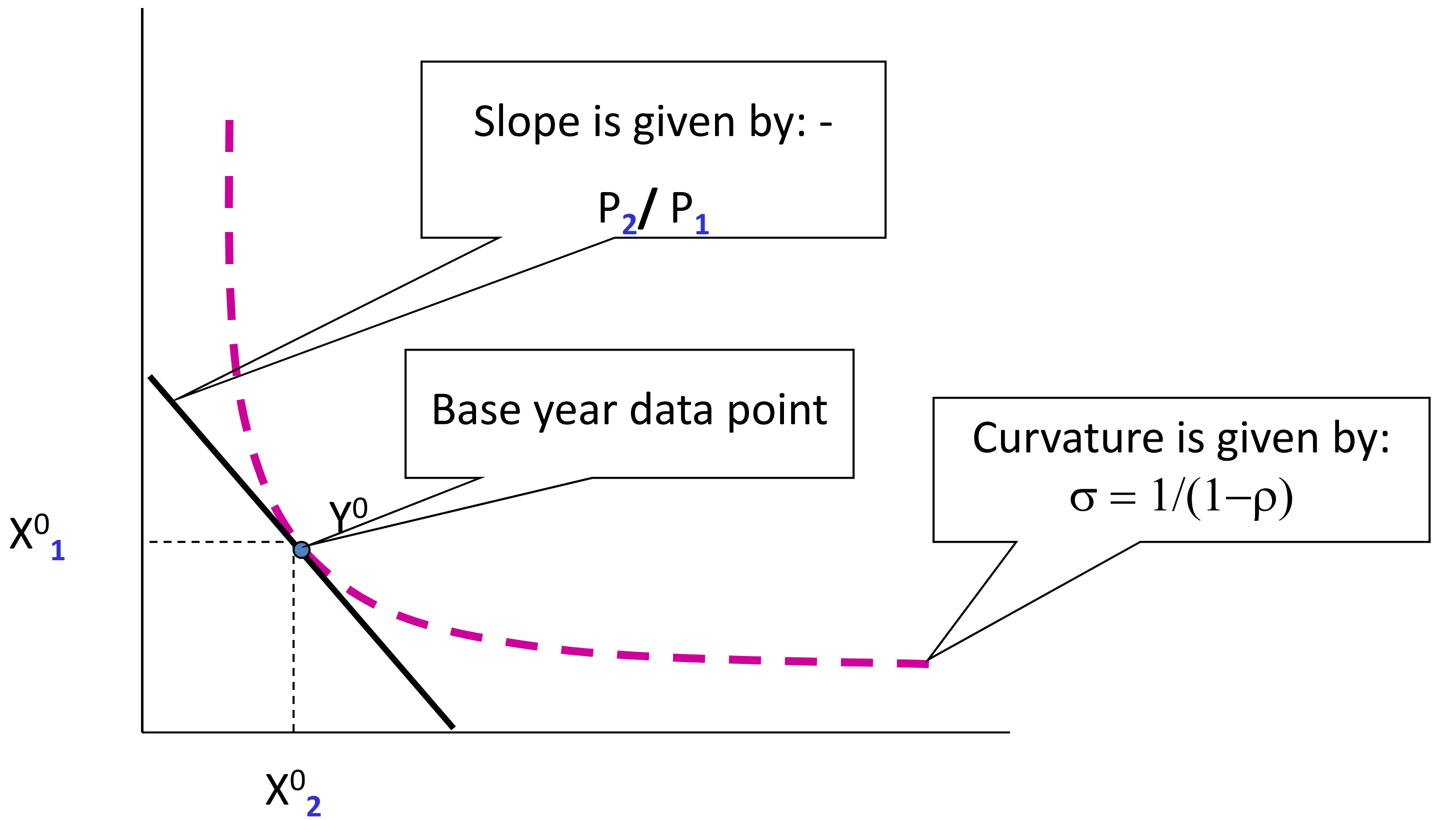
Note that ρ (or substitution parameter σ) is often assumed to be given.

It can then be shown that δ is in fact equal to the (optimal) cost share:

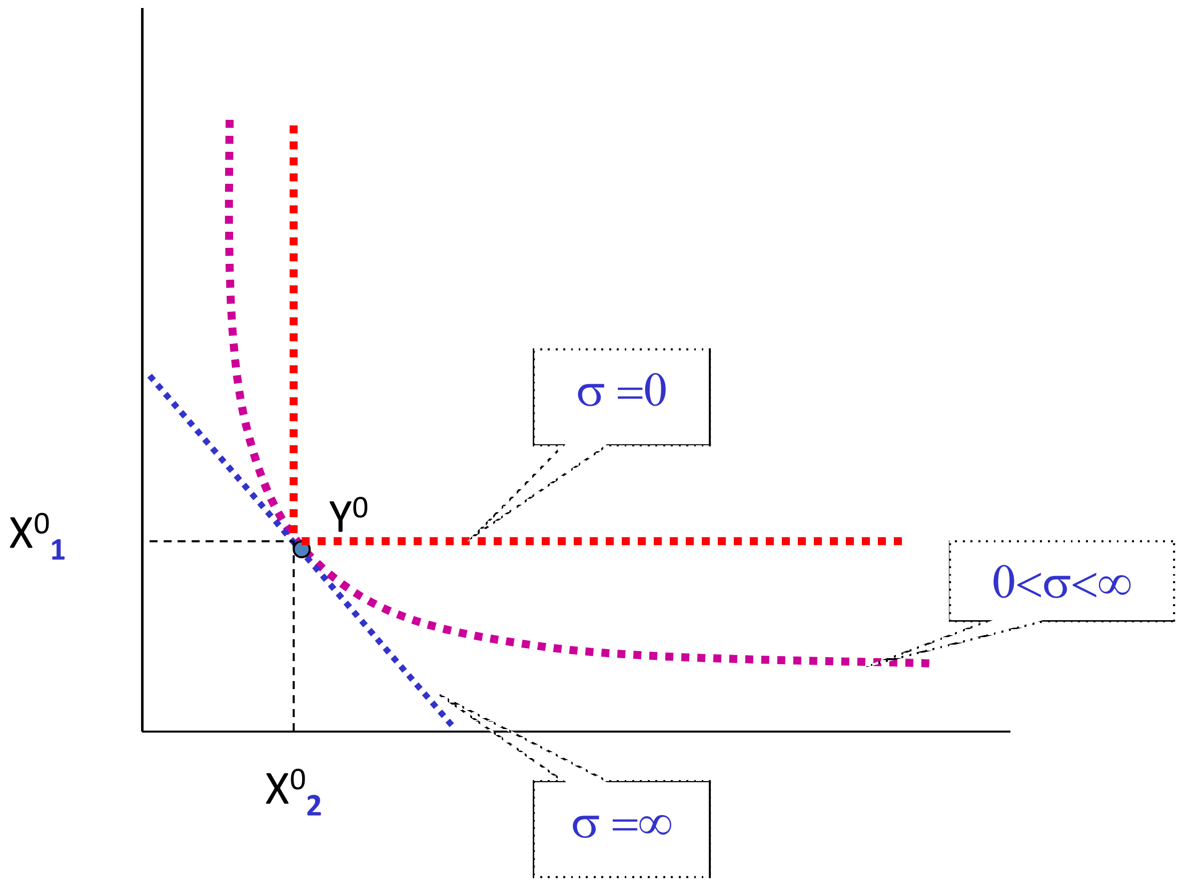
$$\delta_1 = (P^0{}_1 X^0{}_1)/(P^0{}_1 X^0{}_1 + P^0{}_2 X^0{}_2)$$

$$\delta_2 = (P^0{}_2 X^0{}_2)/(P^0{}_1 X^0{}_1 + P^0{}_2 X^0{}_2)$$

Calibration: $Y = \alpha\,[\delta_1 X_1^{\rho} + \delta_2 X_2^{\rho}]^{1/\rho}$



Calibration: $Y = \alpha\,[\delta_1 X_1^{\rho} + \delta_2 X_2^{\rho}]^{1/\rho}$



$\sigma = 1/(1-\rho)$: $\sigma = \infty$ if $\rho = 1$ (perfect substitution);
$\sigma = 1$ if $\rho = 0$ (Cobb-Douglas); $\sigma = 0$ if $\rho = -\infty$ (Leontief);

Treatment of indirect taxes

- Define the power of a tax as the ratio of the price *with* tax over the price without tax; tax rate is then equal to the power of the tax minus 1; tax revenue is equal to the tax base multiplied by the tax rate.
- e.g. If tax base is V1BAS (value of intermediate input at 'basic' price), T1 is the power of the tax on V1BAS, then the value of the good at 'producer' price is V1BAS*T1, tax rate is (T1-1) and tax revenue is V1TAX=V1BAS*(T1-1).

Treatment of indirect taxes

- In general, let VBAS be tax base, T is the power of the tax, and VTAX be the tax revenue. We have: $T=(VBAS+VTAX)/VBAS$
- In absolute level form: $VTAX = VBAS*(T-1)$
- From which derive: $\Delta VTAX = \Delta VBAS*(T-1)+ VBAS* \Delta T$
- Divide by VTAX to get percentage change form:

$$\boldsymbol{vtax} = \boldsymbol{vbas} + (T/(T-1))* \boldsymbol{t}$$

$$\boldsymbol{vtax} = (\boldsymbol{x+p}) + ((VBAS+VTAX)/VTAX)*\boldsymbol{t}$$

or in absolute change form:

$$\Delta VTAX = VTAX*(\boldsymbol{x+p}) + (VBAS+VTAX)*\boldsymbol{t}$$

Change in tax base

Change in tax rate