The Armington Assumption

Short Course on CGE Modeling, United Nations ESCAP

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In standard models of international trade it makes no economic sense to engage in trade in both directions in the same product.

Nonetheless, a characteristic of real world trading patterns is that countries often simultaneously import and export goods in the same product category.

In the applied literature this is accommodated via the Armington assumption. The specification is almost universal in CGE models, so much so that they are often referred to as ‘Armington type’ models.

In the approach, consumers are assumed to have a ‘love of variety’ that generates demand for both domestic and foreign produced products within a product category. Hence, the Armington approach is a special case of the horizontal product differentiation.

In this session we will modify our basic trade model to illustrate the concepts involved.
Session Outline

1. Armington preferences
2. Armington as an activity
3. Implications of the Armington specification
4. Building a GAMS program with Armington
The Armington assumption at its core amounts to making a specific set of assumptions on the structure of preferences.

Consumers are assumed to differentiate between goods based on origin. If we denote consumption of domestically produced goods $d_i$ and imported versions of the same good $m_i$, the utility function is $U = U(d_1, d_2, m_1, m_2)$.

Since goods are defined by how the consumer perceives them, there is nothing new here provided the utility function continues to satisfy the basic axioms — we just have a problem of consumer choice with twice as many arguments as before.

Armington models make use of a nested consumption structure. Final household demand is represented by a standard utility function of the type we have been using, defined across consumption of composite goods. The composites are in turn defined across imported and domestically produced products.
Armington Preferences

Household Demand

Composite 1
Imported 1 Domestic 1

Composite 2
Imported 2 Domestic 2

...
We can write the representative consumer’s utility function as 
\[ U = U(c_1(d_1, m_1), c_2(d_2, m_2)) \]. The functions \( c_i \) are the Armington composite or aggregator functions. They are assumed to be continuous, increasing in both arguments, concave, and homogeneous of degree one. The consumer’s maximization problem is:

\[
\max \quad L = U(c_1(d_1, m_1), c_2(d_2, m_2)) + \lambda [Y - p_1^d d_1 - p_1^m m_1 - p_2^d d_2 - p_2^m m_2]
\]

The first order conditions for a maximum are:

\[
\frac{\partial L}{\partial d_1} = (\frac{\partial U}{\partial c_1})(\frac{\partial c_1}{\partial d_1}) - \lambda p_1^d = 0
\]

\[
\frac{\partial L}{\partial d_2} = (\frac{\partial U}{\partial c_2})(\frac{\partial c_2}{\partial d_2}) - \lambda p_2^d = 0
\]

\[
\frac{\partial L}{\partial m_1} = (\frac{\partial U}{\partial c_1})(\frac{\partial c_1}{\partial m_1}) - \lambda p_1^m = 0
\]

\[
\frac{\partial L}{\partial m_2} = (\frac{\partial U}{\partial c_2})(\frac{\partial c_2}{\partial m_2}) - \lambda p_2^m = 0
\]
The consumer must spend all of their income.
The consumer equates the marginal utility per dollar spent on each good to the marginal utility of income.
We apply the chain rule to convert the units of domestic/importable consumption into units of composite consumption, and then into ‘utility’ units.
At an optimal solution it must be the case that the marginal rate of substitution between any pair of commodities is equal to (minus) the relative price.
Alternative Approach

- The above approach emphasizes the fact that the Armington assumption is really nothing more than the choice of a particular form for the utility function.

- Armington is frequently presented as a two-stage optimization problem. In first stage, the consumer minimizes the expenditure required to generate a unit of the composite good. The minimized cost is the price of the composite.

- We can then solve for the total consumption levels of the composite in terms of the composite prices.

- If you like, you can view this as a competitive industry creating the composite.

- This leaves the upper level utility maximization problem, and its solution, intact. Therefore, the lower level Armington aggregation functions, along with the first order conditions to the lower level problem, can simply be appended to the models that we have developed.
The CES function is generally used as an aggregator. The first order conditions are:

\[
p_i^d = p_i c_i [\delta_i^A d_i^{\rho_i^A} + (1 - \delta_i^A) m_i^{\rho_i^A}]^{-1} \delta d_i^{\rho_i^A - 1}
\]

\[
p_i^m = p_i c_i [\delta_i^A d_i^{\rho_i^A} + (1 - \delta_i^A) m_i^{\rho_i^A}]^{-1} (1 - \delta_i^A) m_i^{\rho_i^A - 1}
\]

\[
c_i = \gamma_i^A [\delta_i^A d_i^{\rho_i^A} + (1 - \delta_i^A) m_i^{\rho_i^A}]^{1/\rho_i^A}
\]

These three equations determine the three new unknowns (for each industry) that we are introducing to the model, and can be incorporated directly.
Cautions

- The use of CES functions for Armington aggregation has some serious implications.
- Assuming that a good is initially consumed in both its domestic and imported versions, it will continue to be consumed in both versions even when the economic system is shocked.
- In practice we do not observe the large swings in trade and production that tend to be associated with standard textbook general equilibrium models. However, the feature cuts in both directions.
- Suppose that in the initial equilibrium the level of consumption of the imported version of a good is zero, so that consumption of the composite is equivalent to consumption of the domestically produced good. Even if the price of the imported version of goods falls in a simulation, imports will remain at zero.
Starting with a small economy model, add in the new parameters, \( \text{RHO}_A(I), \text{DELTA}_A(I), \text{GAMMA}_A(I) \).

Assign new parameters for the prices of domestically produced and imported goods, \( \text{PD}(I) \) and \( \text{PM}(I) \).

Add parameters to hold the initial values \( \text{DO}(I), \text{MO}(I), \) and \( \text{PO}(I) \).

Normalize all prices and provide consistent base data. Note that exports \( \text{XO}(I) \) must equal production \( \text{QO}(I) \) less consumption of domestic goods \( \text{DO}(I) \), composite consumption \( \text{CO}(I) \) must equal consumption of domestic goods \( \text{DO}(I) \) plus consumption of imported goods \( \text{MO}(I) \).
- Set values for the substitution parameters.
- Calibrate the other parameters of the Armington functions (this is essentially the same as calibrating the CES production function).
- Set levels and bounds.
- Add the new equations corresponding to the conditions derived above.
- Adjust the material balance equations.
- Set up a model statement and check.
Can you build a large country model with Armington preferences? (Hint: The usual convention is to fix the price of imports, but allow the world price of exports to vary.)

How would you go about adding taxes on trade and other activities to this model? Are import and export taxes still symmetric? In what sense?
The original reference is Armington (1969).
Lloyd and Zhang (2006) provide a detailed comparison of the Armington approach with the standard trade model.
The review of Kehoe (2003) is useful to better understand the consequences of the specification.
Recent empirical estimates of the size of the Armington elasticities can be found in Hertel et al. (2007).
Modeling issues are discussed in more detail Gilbert and Tower (2013), chapter 23.