

Modeling Demand

Short Course on CGE Modeling, United Nations ESCAP

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- In CGE models, final demand is derived from the utility maximization problem of a representative household (or in some cases households).
- This is a very familiar problem, but working through it carefully will provide us with a number of insights into how CGE-type models are constructed in practice.
- In this session we will review the basic consumer problem, work through an example, and finally implement the model in GAMS.

- 1 The utility maximization problem
- 2 Example using Cobb-Douglas
- 3 Building the model in GAMS
 - Setting up the model
 - Calibration
 - Simulation and testing

Utility Maximization

- Consider a consumer that has preferences satisfying the axioms of consumer choice, and where their preferences can be summarized by the utility function $U = U(c_1, c_2)$, where c_i is consumption of the i th good.
- The usual properties apply to the utility function (i.e., continuity, monotonicity, and quasi-concavity).
- The consumer choice problem can be viewed as choosing c_1 and c_2 such that the consumer maximizes $U = U(c_1, c_2)$ subject to the budget constraint $Y = p_1c_1 + p_2c_2$, where Y is money income.
- We assume an internal solution for simplicity.

Formal Problem

The problem can be written:

$$\max U(c_1, c_2) \quad \text{s.t.} \quad Y = p_1 c_1 + p_2 c_2$$

or equivalently:

$$\max \mathcal{L} = U(c_1, c_2) + \lambda[Y - p_1 c_1 - p_2 c_2]$$

The first order conditions are:

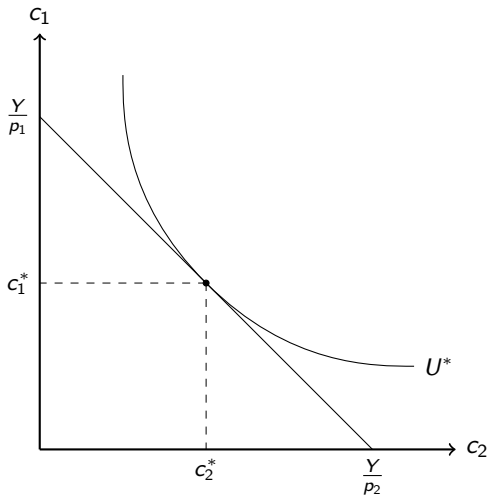
$$\partial \mathcal{L} / \partial c_1 = \partial U / \partial c_1 - \lambda p_1 = 0$$

$$\partial \mathcal{L} / \partial c_2 = \partial U / \partial c_2 - \lambda p_2 = 0$$

$$\partial \mathcal{L} / \partial \lambda = Y - p_1 c_1 - p_2 c_2 = 0$$

- The solution to the maximization problem is the simultaneous solution to these three equations for c_1 , c_2 and λ , expressed in terms of p_1 , p_2 and Y .
- At an optimum, the consumer will spend all income, and the marginal utility per dollar spent on each good must equal the marginal utility of income (λ).
- Another way of interpreting the conditions is that the money value of the utility generated by the last unit of each good purchased must equal its price.
- Solving explicitly for the optimal consumption bundles yields the Marshallian demand functions.

Geometric Interpretation



Example - Cobb-Douglas Utility

Suppose that the utility function takes the form $U = \alpha c_1^\beta c_2^{1-\beta}$, where $0 < \beta < 1$. The Marshallian demands will be:

$$c_1 = \beta Y / p_1$$
$$c_2 = (1 - \beta) Y / p_2$$

Set Notation

GAMS is a set based language, a feature that allows us to build models of arbitrary dimensions easily. The underlying set for the consumer choice problem is the set of goods the consumer can choose, $\mathbf{I} = \{1, 2\}$. We can write the Marshallian demands as:

$$c_i = \beta_i Y / p_i \quad \forall i \in \mathbf{I}$$

Each exponent is a separate share, so to be equivalent to our original representation we need $\sum_{\forall i \in \mathbf{I}} \beta_i = 1$. The utility function can be rewritten in a similar fashion:

$$U = \alpha \prod_{\forall i \in \mathbf{I}} c_i^{\beta_i}$$

This way of writing the model is more compact. It is also easier to expand.

Now let us consider exactly how the problem can be expressed in the GAMS language.

Our first task is to create a set which will index the goods:

```
SET I Goods /1,2/;
```

The keyword is followed by an arbitrary name for the set, I, an optional description, then the elements of the set enclosed in forward slashes and separated by commas. The names used for set elements are also arbitrary. The command is completed with a semicolon.

Next, we need to define labels for all of the parameters and exogenous variables in the model. We are also going to define labels for the initial values of our endogenous variables:

PARAMETERS

ALPHA	Shift parameter in utility
BETA(I)	Share parameters in utility
Y	Income
P(I)	Prices
U0	Initial utility level
CO(I)	Initial consumption levels;

The keyword `PARAMETERS` is followed by a list of parameter names, indexed as appropriate, and their descriptions.

Our next task is to assign names for the variables:

```
VARIABLES
```

```
U           Utility level
```

```
C(I)       Consumption levels;
```

The keyword `VARIABLES` is followed by a list of variable names, indexed as appropriate, and their descriptions.

GAMS Program - Equations

We enter names for equations in the model in the same way that we enter names for parameters and variables:

```
EQUATIONS  
UTILITY    Utility function  
DEMAND(I)  Demand functions;
```

The keyword is EQUATIONS, followed by a list of names, which are also indexed, with optional descriptions. We define the structure of the equations in terms of the variables and parameters:

```
UTILITY..U=E=ALPHA*PROD(I, C(I)**BETA(I));  
DEMAND(I)..C(I)=E=BETA(I)*Y/P(I);
```

These are the GAMS equivalents of the equations we derived.

With the labels created, we can assign some suitable values to the parameters:

```
P(I)=1;  
Y=100;  
CO(I)=50;  
UO=Y;  
BETA(I)=P(I)*CO(I)/Y;  
ALPHA=UO/PROD(I, CO(I)**BETA(I));
```

The process is called **calibration** – which is essentially solving the model for the parameter values consistent with the desired solution.

GAMS Program - Initial Levels and Bounds

Since the values of the variables are determined by the model, we cannot assign values to them as such. But, we can give GAMS starting values for the variables. We use the initial values we assigned/calculated:

```
U.L=U0;
```

```
C.L(I)=C0(I);
```

We can also assign logical bounds on the values the variables can take:

```
C.L0(I)=0;
```

The last stage is to tell GAMS which of our equations constitute the model, in this case all of them, and then run a test solution:

```
MODEL UMAX /ALL/;  
SOLVE UMAX USING NLP MAXIMIZING U;
```

The first statement says that the model that we will call UMAX consists of all the equations (i.e., it is equivalent to MODEL UMAX /UTILITY, DEMAND/).

- We have now built our first simple numerical simulation model. Once we are satisfied that the model is correctly replicating the solution, we can examine the effects of changes in the economic situation by altering the values of parameters and/or exogenous variables and executing another SOLVE command.
- Some things to try:
 - Changing prices and/or income in various combinations.
 - Altering the model parameters.
 - Experimenting with different utility function forms.
 - Solving the corresponding expenditure minimization problem.
 - Constructing alternative measures of welfare changes.

- Most of this material is covered in intermediate microeconomics texts such as Varian (2009).
- Dixit (1990) is a good introduction to optimization.
- This GAMS example is developed more fully in Gilbert and Tower (2013), chapter 3.