Modeling Supply

Short Course on CGE Modeling, United Nations ESCAP

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Introduction

- Now that we have modeled the firm's production decision, we will complete a basic model of supply.
- We will consider two cases. In the first we will assume that all factors of production are mobile between economic activities.
- Next we will show how exception handling can be used to introduce specific factors.

Session Outline

1. The long-run production problem
2. Building the model in GAMS
3. Extending to allow specific factors

GDP Maximization

- Consider the problem of maximizing the value of total output (GDP), at given prices, subject to the constraints imposed by resource limitations.
- This sounds like a social planning problem and may in fact be viewed as such, it is not necessary to do so. When there are no factor market distortions, factor endowments are fixed, and competition prevails, the market maximizes the value of output at given output prices.
- We will start with the two factor, two good case.
- Both factors are assumed to be mobile.

Formal Problem

The problem can be written:

$$\max \quad \mathscr{L} = p_1 q_1(K_1, L_1) + p_2 q_2(K_2, L_2) + \lambda[\bar{K} - K_1 - K_2] + \mu[\bar{L} - L_1 - L_2]$$

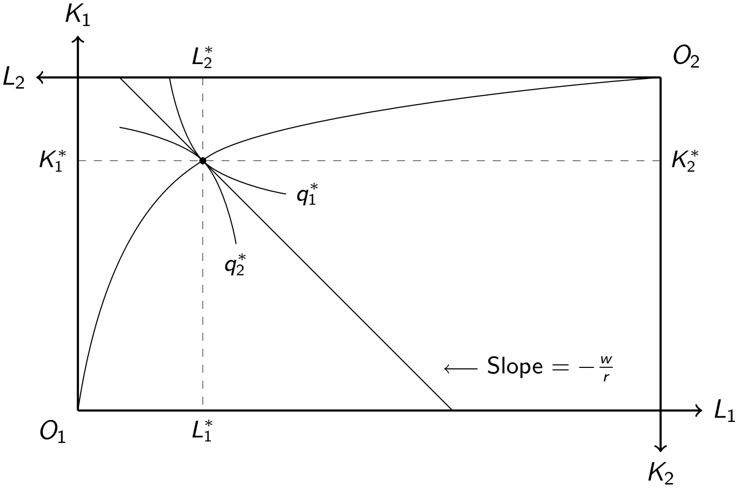
The first order conditions are:

$$\partial \mathscr{L} / \partial K_1 = p_1 \partial q_1 / \partial K_1 - \lambda = 0$$
$$\partial \mathscr{L} / \partial L_1 = p_1 \partial q_1 / \partial L_1 - \mu = 0$$
$$\partial \mathscr{L} / \partial K_2 = p_2 \partial q_2 / \partial K_2 - \lambda = 0$$
$$\partial \mathscr{L} / \partial L_2 = p_2 \partial q_2 / \partial L_2 - \mu = 0$$
$$\partial \mathscr{L} / \partial \lambda = \bar{K} - K_1 - K_2 = 0$$
$$\partial \mathscr{L} / \partial \mu = \bar{L} - L_1 - L_2 = 0$$

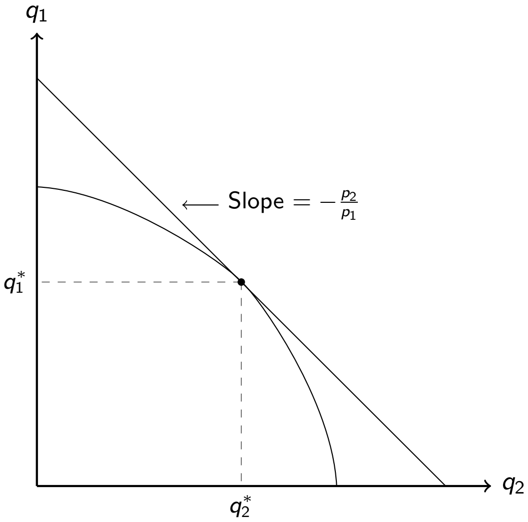
Interpretation

- Under competitive conditions, the shadow values on the resource constraints are factor prices.
- Hence, at an optimum, each factor price is equal to the value of the marginal product of that factor in each industry.
- Also at an optimum, resources are fully utilized.

Geometric Interpretation



Geometric Interpretation



GAMS Program - Sets

To build the model in GAMS, we can modify the firm's problem:

We'll create two sets which will index the goods and factors:

```
SET I Goods /1,2/;
SET J Factors /K,L/ ;
ALIAS (J, JJ);
```

GAMS Program - Parameters

Now we'll define the parameters, extending the dimensions over the goods and adding in new holders for endowments and GDP:

```
PARAMETERS
GAMMA(I)    Shift parameter in production
DELTA(J,I)  Share parameters in production
RHO(I)      Elasticity parameter in production
QO(I)       Output level
RO(J)       Factor prices
FO(J,I)     Initial factor use levels
FBAR(J)     Initial endowments
P(I)        Prices
GDPO        Gross domestic product;
```

GAMS Program - Variables

Our next task is to assign names for the variables:

```
VARIABLES
Q(I)      Output levels
R(J)      Factor prices
F(J,I)    Factor use levels
GDP       Gross domestic product;
```

Notice that Q and R are now endogenous not exogenous.

GAMS Program - Equations

We enter names for equations in the model:

```
EQUATIONS
PRODUCTION(I)   Production functions
RESOURCE(J)     Resource constraints
FDEMAND(J,I)    Factor demand functions
INCOME          Gross domestic product;
```

GAMS Program - Equations

Then we define the structure of the equations in terms of the variables and parameters:

```
PRODUCTION(I)..Q(I)=E=GAMMA(I)*SUM(J, DELTA(J,I)*F(J,I)**
               RHO(I))**(1/RHO(I));
RESOURCE(J)..FBAR(J)=E=SUM(I, F(J,I));
FDEMAND(J,I)..R(J)=E=P(I)*Q(I)*SUM(JJ, DELTA(JJ,I)*F(JJ,I)**
               RHO(I))**(-1)*DELTA(J,I)*F(J,I)**(RHO(I)-1);
INCOME..GDP=E=SUM(I, P(I)*Q(I));
```

These are the GAMS equivalents of the equations we derived.

GAMS Program - Final Steps

- To complete the program the steps are the same as in the previous examples.
- We need to calibrate.
- Then assign the levels and lower bounds.
- Finally, we set up a model statement and a solve statement.
- These steps are much the same as in the previous examples.

Extensions - Specific Factors

- If we are interested in relatively short-run responses to economic shocks, we might want to treat capital as specific to each industry.
- It turns out that we can accomplish this very efficiently in GAMS using a feature called exception handling.
- Exception handling is used to restrict the range/domain over which expressions are evaluated.

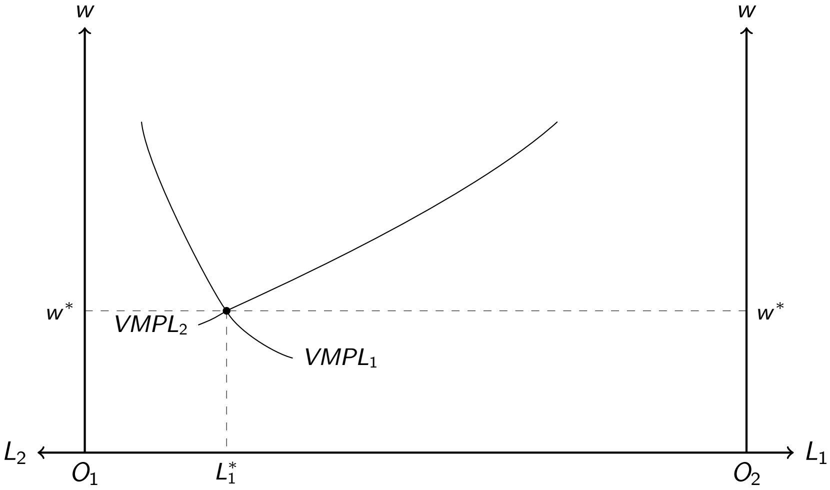
GDP Maximization with Specific Capital

The problem can be written:

$$\begin{aligned} \max \quad \mathscr{L} &= p_1 q_1(K_1, L_1) + p_2 q_2(K_2, L_2) + \lambda_1[\bar{K}_1 - K_1] \\ &+ \lambda_2[\bar{K}_2 - K_2] + \mu[\bar{L} - L_1 - L_2] \end{aligned}$$

The new element is the separation of capital into two types, each of which will have a different price. I'll leave the derivation of the first order conditions as an exercise (they are very similar to the example above).

Geometric Interpretation



GAMS Implementation

- Starting with the production model we have built, extend the dimension of the factor set, `SET J Factors /K,L,N/;`
- Assign a zero value to factor demands for `N` in industry 1, and `K` in industry 2.
- Alter the production and factor demand functions to read:

```
PRODUCTION(I)..Q(I)=E=GAMMA(I)*SUM(J$FO(J,I), DELTA(J,I)*
   F(J,I)**RHO(I))**(1/RHO(I));
FDEMAND(J,I)$FO(J,I)..R(J)=E=P(I)*Q(I)*SUM(JJ$FO(JJ,I),
   DELTA(JJ,I)*F(JJ,I)**RHO(I))**(-1)*DELTA(J,I)*F(J,I)
   **(RHO(I)-1);
```

- Alter the calibration of `DELTA` and `GAMMA` in the same way.

Extensions - Higher Dimensions

- It is possible to build a higher dimensional model by extending the dimensions of the underlying sets and providing appropriate equilibrium data.
- If we do so, we need to be careful about corner solutions (if prices are given).
- We might also need to introduce exception handling if some factors are not used in all sectors (as in the specific factors model).

Summing Up

- Building the model of production is a major step — we have now completed our first general equilibrium model of the production side of an economy.
- Although the model is small scale, its dimensions can easily be extended.
- Understanding how the model responds to changes in the economic environment is critical to understanding what goes on inside larger CGE models, the basic mechanisms are the same.

Exercises

- Numéraire shock.
- Changing prices and the Stolper-Samuelson theorem.
- Changing endowments and the Rybczynski theorem.
- Changing technology.
- Comparing long and short-run economic responses.

Further Reading

- A good treatment of the HOS model and its characteristics is Bhagwati *et al*. (1998).
- After working through their exposition, you may find it rewarding to revisit the classic articles by Stolper and Samuelson (1941) and Rybczynski (1955).
- The classic reference for the specific factors model is Jones (1971).
- The GAMS examples are developed fully in Gilbert and Tower (2013), chapters 5 and 6.