Session 1: The Theoretical Gravity Model

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ARTNeT Capacity Building Workshop for Trade Research: “Behind the Border” Gravity Modeling

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Outline

1. Introduction

2. The Theoretical Gravity Model
   - Gravity with Gravitas
   - Empirical Implications of Gravity Theory

3. Fixed Effects Estimation

4. Random Effects Estimation

5. Summary
The basic gravity model provides a respectable place to start.

But if we look more closely, we will find that it has some unattractive implications from an economic point of view.

Doing some theory allows us to reformulate the gravity model in much more attractive way.
The Relative Price Problem

The Basic Gravity Model

\[
\log (X_{ij}) = b_0 + b_1 \log (Y_i) + b_2 \log (Y_j) + b_3 \log (d_{ij}) + e_{ij}
\]

- Yesterday, we interpreted \( b_3 = -1 \) as indicating that a 1% increase in bilateral distance (transport costs) is associated with a 1% decrease in bilateral trade.

- In fact, this presents some serious problems in the context of a world with many countries.
The Relative Price Problem

The Basic Gravity Model

\[ \log (X_{ij}) = b_0 + b_1 \log (Y_i) + b_2 \log (Y_j) + b_3 \log (d_{ij}) + e_{ij} \]

- Do trade flows between i and j only depend on bilateral trade costs, without any adjustment for the level of trade costs prevailing on other routes?
- If trade costs fall between i and j, will trade flows with all other countries remain unchanged?
- If trade costs on all routes fall by the same proportion, will trade everywhere increase by the same proportion?
To try and fix these problems, it makes sense to go back to fundamentals.
The basic gravity model picks up some important empirical regularities, but has been posited without any explicit theoretical foundation.
If we add in some micro-foundations, hopefully we will be able to derive something that looks a lot like gravity, but deals with the relative cost problem.
A number of papers develop solid theoretical bases for the gravity model.

We will focus on the “gravity with gravitas” model set out by Jim Anderson and Eric Van Wincoop in the AER (2003) and JEL (2004).

It is treated by many (most?) applied trade researchers as the empirical baseline.

Build on it by all means, but ignore it at your peril!
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To derive the AvW model, we proceed as follows (see handout):

1. Set out a consumption side based on love of variety preferences;
2. Set out a production side with large group monopolistic competition;
3. Introduce trade costs, and relate domestic and foreign prices;
4. Impose some macro identities and aggregate to produce a gravity-like model.
The Theoretical Gravity Model

Consumption Side

Love of Variety Preferences

\[ U_i = \sum_{k=1}^{K} \left\{ \int_{v \in V_i^k} \left[ x_i^{k} (v) \right]^{1 - \frac{1}{\sigma_k}} dv \right\}^{\frac{1}{1 - \frac{1}{\sigma_k}}} \]

- C countries (i), K sectors (k), each with measure V varieties (v).
- Intrasectoral (between varieties) elasticity of substitution \( \sigma_k \).
- Can easily be varied: one sector only, Cobb-Douglas first tier, etc.
- Utility increases with greater consumption of each variety, and consumption of more varieties.
The Theoretical Gravity Model

Production Side

Constant Markup Pricing

\[ p_i^k (v) = \left( \frac{\sigma_k}{\sigma_k - 1} \right) w a_i^k \]

- Each firm makes a unique variety under increasing returns to scale. Marginal cost \( a \) is constant by country-sector.
- With a very large number of firms, each one of them takes the overall price level as given.
- This allows us to shut down strategic interactions: all firms in a sector price at the same, constant markup over marginal cost.
International trade is costly: if I want one unit to arrive, I must ship $\tau_{ij} \geq 1$ units.

Thus, the price of a variety produced in one country and consumed in another is increased by the same factor over the price in the home country.

Think of it like an ad valorem tariff, or variable transport costs. No fixed costs of market entry.
The Theoretical Gravity Model

Aggregate and use Macro-Identities to get Gravity with Gravitas

The AvW Gravity Model

\[
X_{ij}^k = \frac{Y_i^k E_j^k}{Y_i^k} \left\{ \frac{\tau_{ij}^k}{\prod_i^k P_j^k} \right\}^{1-\sigma_k}
\]

\[
\left( \prod_i^k \right)^{1-\sigma_k} = \sum_{j=1}^C \left\{ \frac{\tau_{ij}^k}{P_j^k} \right\}^{1-\sigma_k} \frac{E_j^k}{Y_i^k}
\]

\[
\left( P_j^k \right)^{1-\sigma_k} = \sum_{j=1}^C \left\{ \frac{\tau_{ij}^k}{\prod_i^k} \right\}^{1-\sigma_k} \frac{Y_i^k}{Y_i^k}
\]

- Bilateral trade depends positively on the size of the importing and exporting countries, but negatively on trade costs.
- The two price indices capture the fact that it is relative trade barriers that matter.
The AvW Gravity Model

\[
X_{ij}^k = \frac{Y_i^k E_i^k}{Y^k} \left\{ \frac{\tau_{ij}^k}{\Pi_{ij}^k P_j^k} \right\}^{1-\sigma_k}
\]

\[
(\Pi_{ij}^k)^{1-\sigma_k} = \sum_{j=1}^{C} \left\{ \frac{\tau_{ij}^k}{P_j^k} \right\}^{1-\sigma_k} \frac{E_j^k}{Y^k}
\]

\[
(P_j^k)^{1-\sigma_k} = \sum_{i=1}^{C} \left\{ \frac{\tau_{ij}^k}{\Pi_{ij}^k} \right\}^{1-\sigma_k} \frac{Y_i^k}{Y^k}
\]

- $\Pi_{ij}^k$ is outward multilateral resistance.
- Exports from i to j depend on bilateral trade costs, but also on trade costs affecting i’s exports to all other markets.
The Theoretical Gravity Model

\( X_{ij}^k = \frac{Y_i^k E_j^k}{Y^k} \left\{ \frac{\tau_{ij}^k}{\prod_i^k P_j^k} \right\}^{1-\sigma_k} \)

\((\prod_i^k)^{1-\sigma_k} = \sum_{j=1}^C \left\{ \frac{\tau_{ij}^k}{P_j^k} \right\}^{1-\sigma_k} \frac{E_j^k}{Y^k} \)

\((P_j^k)^{1-\sigma_k} = \sum_{i=1}^C \left\{ \frac{\tau_{ij}^k}{\prod_i^k} \right\}^{1-\sigma_k} \frac{Y_i^k}{Y^k} \)

- \( P_j^k \) is inward multilateral resistance.
- Exports from i to j depend on bilateral trade costs, but also on trade costs affecting j’s imports from all other markets.
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5. Summary
AvW looks like gravity, but also has some important differences with respect to the basic model.

- Inclusion of the multilateral resistance terms.
- Selection of variables.
- Interaction between trade costs and the substitution elasticity.

What are the implications of these differences for empirical work?
Theory-Consistent Gravity Data
Trade Data

The AvW Gravity Model

\[
X_{ij}^k = \frac{Y_i^k E_j^k}{Y^k} \left\{ \frac{\tau_{ij}^k}{\Pi_i^k P_j^k} \right\}^{1-\sigma_k}
\]

\[
(\Pi_i^k)^{1-\sigma_k} = \sum_{j=1}^{C} \left\{ \frac{\tau_{ij}^k}{P_j^k} \right\}^{1-\sigma_k} \frac{E_j^k}{Y^k} ; \left( P_j^k \right)^{1-\sigma_k} = \sum_{i=1}^{C} \left\{ \frac{\tau_{ij}^k}{\Pi_i^k} \right\}^{1-\sigma_k} \frac{Y_i^k}{Y^k}
\]

- The AvW gravity model needs trade data in nominal value terms.
- Each observation should represent a unidirectional flow between a pair of countries, e.g. exports from i to j, not total trade in both directions (i to j or j to i), or the average, etc.
The Theory-Consistent Gravity Data

GDP Data

The AvW Gravity Model

\[ X_{kj}^k = \frac{Y_k^i E_j^k}{Y^k} \left\{ \frac{\tau_{ij}^k}{\prod_i^k \prod_j^k} \right\}^{1-\sigma_k} \]

\[ (\prod_i^k)^{1-\sigma_k} = \sum_{j=1}^{C} \left\{ \frac{\tau_{ij}^k}{P_j^k} \right\}^{1-\sigma_k} \left( \frac{E_j^k}{Y^k} \right)^{1-\sigma_k} = \sum_{i=1}^{C} \left\{ \frac{\tau_{ij}^k}{\prod_i^k} \right\}^{1-\sigma_k} \frac{Y_i^k}{Y^k} \]

- GDP should be in nominal, not real, terms.
- When the model is estimated sector by sector, we would ideally like sectoral expenditures and production, not economy-wide GDP.
We need to do something to take account of multilateral resistance.

Standard price indices (CPI, PPI, etc.) are not aggregated in the way implied by theory, and so can at best be a poor proxy for the true (ideal) price indices we need.
The Trade Costs Function

The model depends on trade costs $\tau_{ij}$, but they are not directly observable as an aggregate. We need to build them up by specifying them in terms of observables.

Most commonly, the trade costs function is specified as follows:

$$\ln \left( \frac{\tau_{ij}^k}{Y_i} \right) = t_1 \ln (d_{ij}) + t_2 \ln \text{lang}_{ij} + t_3 \ln (1 + \text{tariff}_{ij}) + t_4 \text{border}_{ij} + \ldots$$
Taking Account of Multilateral Resistance

The AvW Gravity Model

\[
\ln \left( X_{ij}^k \right) = \\
\ln \left( Y_i^k \right) + \ln \left( E_j^k \right) - \ln \left( Y^k \right) + (1 - \sigma_k) \left[ \ln \tau_{ij}^k + \ln \Pi_i^k + \ln P_j^k \right]
\]

- The basic gravity model gets the first four terms approximately correct, but leaves out the last two.
- Unless the MR terms have zero correlation with exports and trade costs (or GDP), then we have omitted variables bias.
How serious is this OV bias empirically?

Distance coefficient from the original gravity model without fixed effects = -1.277***.

Distance coefficient from the fixed effects gravity model = -1.596***.

The difference between these estimates is statistically significant at the 1% level. Is it economically significant?
Taking Account of Multilateral Resistance

What to Do?

The AvW Gravity Model

\[ X_{ij}^k = \ln (Y_i^k) + \ln (E_j^k) - \ln (Y^k) + (1 - \sigma_k) \left[ \ln \tau_{ij}^k + \ln \Pi_i^k + \ln P_j^k \right] \]

- The problem is easier stated than solved, because there is no way to observe \( \ln \Pi_i^k + \ln P_j^k \).
- We cannot just find some data, include it, and fix the OV bias!
Using Dummies to Capture Multilateral Resistance

The AvW Gravity Model–Aggregate Data

\[ X_{ij} = \ln(Y_i) + \ln(E_j) - \ln(Y) + (1 - \sigma) \left[ \ln\tau_{ij} + \ln\Pi_i + \ln P_j \right] \]

- The term \(-\ln(Y)\) is common across all exporters and importers; thus, it can be captured through a constant in the regression model.
- The term \(\ln(E_j) + \ln P_j\) is constant across all importers for a given exporter; thus, it can be captured through an importer dummy variable (fixed effect).
- The term \(\ln(Y_i) + \ln\Pi_i\) is constant across all exporters for a given importer; thus, it can be captured through an exporter dummy variable (fixed effect).
Using Dummies to Capture Multilateral Resistance

Advantages of Dummy Variables

- An aggregate gravity model with a constant, and dummy variables for each exporter and each importer will therefore take proper account of multilateral resistance, and should produce unbiased estimates.
- Very simple to estimate, but takes account of some sophisticated effects.
- NxN observations, but N+N dummies; degrees of freedom are usually sufficient.
Using Dummies to Capture Multilateral Resistance

Disadvantages of Dummy Variables

- Dimensionality quickly becomes an issue with sectoral models: N+N can be in the hundreds, or thousands.
- Because of collinearity constraints, we cannot identify separate effects due to factors that vary in the exporter or importer dimensions. Only factors varying bilaterally can be identified.
Using Dummies to Capture Multilateral Resistance

- Estimation using panel data techniques can make it possible to reduce the dimensionality problem somewhat, but it remains an issue in large/detailed datasets.

- To deal with the collinearity problem, variables can sometimes be transformed so as to vary by country pair:
  - Sum of exporter and importer values.
  - Average of exporter and importer values, etc.

- Try to go back to theory to see if this is an appropriate thing to do in a given circumstance.
As suggested previously, things get even more complicated with sectoral gravity models. Dummy variables need to be specified in the importer-sector, exporter-sector, and sector dimensions, because:

$$\ln \left( X_{ij}^k \right) = \ln \left( Y_i^k \right) + \ln \left( E_j^k \right) - \ln \left( Y^k \right) + (1 - \sigma_k) \left[ \ln \tau_{ij}^k + \ln \Pi_i^k + \ln P_j^k \right]$$

In addition, trade costs need to be interacted with sector dummies in order to take account of varying elasticities of substitution across sectors.
Depending on the level of sectoral disaggregation used, this approach can result in huge numbers of parameters. Models can take a long time, and a big computer, to estimate. It is usually much easier to estimate separate models for each sector.
Option 1: enter the dummies manually and use OLS:
- `tab importer, gen(imp_dum_*)`
- `reg ln_trade ... imp_dum_*, robust`

Option 2: use a panel estimator (OLS + a trick) to account for one set of dummies:
- `iis importers`
- `xtreg ln_trade ..., robust fe`
Random Effects: An Alternative to Dummies

- Fixed effects (dummy variables) are one way of accounting for unobserved heterogeneity across countries, in this case due to multilateral resistance.

- A common alternative in the econometrics literature is random effects:
  - Fixed effects allow for free or structureless variation;
  - Random effects require that unobserved heterogeneity obey some probability constraints, i.e. follow a particular distribution.
Random Effects: An Alternative to Dummies

Advantages of Random Effects

- The dimensionality constraint is greatly relaxed: even very large models can be estimated relatively quickly.
- Allows inclusion of variables (like GDP) that vary in the same dimension as the random effects.
- Simple to estimate single-dimensional RE models in Stata:
  - iis (importers)
  - xtreg ln_trade ln_gdp_imp [etc.], re robust.
Disadvantages of Random Effects

- Random effects rely on a strong assumption: multilateral resistance is normally distributed across countries, with a given standard deviation.
- The AvW model tells us that multilateral resistance is important, but it doesn’t tell us anything about its distribution.
- In practice, compare RE and FE estimates.
Random Effects: An Alternative to Dummies

- Distance coefficient from the original gravity model without fixed effects = -1.277***.
- Distance coefficient from the fixed effects gravity model = -1.596***.
- Distance coefficient from the random effects (importer) gravity model = -1.313***
  - Not statistically different from OLS, but significantly different from fixed effects. CAUTION!
Introduction
The Theoretical Gravity Model
Fixed Effects Estimation
Random Effects Estimation
Summary
However, gravity models now need to account for multilateral resistance—otherwise, there is a high risk of biased estimates.

The most common approach to the aggregate gravity model is to use fixed effects (dummy variables) by importer and by exporter.

Random effects can also be used, but they rely on a stronger—and possibly invalid—assumption.

For sectoral gravity models, the simplest approach is to estimate separately, sector by sector.