Bonus Vetus OLS: A Simple Method for Approximating International Trade-Cost Effects using the Gravity Equation

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Abstract

Motivated to solve the "border puzzle" of Canadian-U.S. trade, theoretical foundations for the gravity equation of international trade were enhanced recently to emphasize the need to account for the endogeneity of multilateral price (resistance) terms, cf., Anderson and van Wincoop (2003). While region-specific fixed effects can also generate consistent estimates of gravity-equation coefficients, Anderson and van Wincoop argue that proper computation of general equilibrium comparative statics requires custom estimation of the entire system of trade-flow and nonlinear price equations that explain bilateral trade flows. Unfortunately, since Anderson and van Wincoop (2003) most empirical researchers using the gravity equation have ignored this seminal concern. Moreover, in some circumstances, nonlinear techniques to incorporate the influence of multilateral price terms simply are feasible and an alternative approach is needed.

In this paper, we discuss such circumstances and suggest an alternative approach. The key is using a first-order log-linear Taylor-series expansion to approximate the multilateral price terms; effectively, we decompose the influences of multilateral resistance and of nonlinearity. Among several findings, we note three. First, the Taylor-series expansion allows us to solve for a simple log-linear gravity equation revealing a transparent – and pedagogically useful – theoretical relationship among bilateral trade flows, regional and world incomes, and bilateral, multilateral, and world trade costs. Second, we provide empirical and Monte Carlo econometric results supporting that virtually identical coefficient estimates can be obtained easily by estimating a reduced-form gravity equation including theoretically-motivated exogenous trade-cost terms. Third, we show that our methodology generalizes to many settings and delineate the economic conditions under which our approach works well for computing comparative statics and under which it does not. In a Monte Carlo analysis of a typical gravity equation using 88 countries, we show that the approximation errors are less than five percent in *74 percent* of the 3872 country pairings. Moreover, we show that the MR terms increase the most for small *and close* countries, and we use a "fixed-point" iteration method to demonstrate that economic size *relative to bilateral distance* can alone explain systematically the approximation errors.

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1. Introduction

For nearly a half century, the gravity equation has been used to explain econometrically the *ex post* effects of economic integration agreements, national borders, currency unions, immigrant stocks, language, and other measures of "trade costs" on bilateral trade flows. Until recently, researchers typically focused on a simple specification akin to Newton's Law of Gravity, whereby the bilateral trade flow from region i to region j was a multiplicative (or log-linear) function of the two countries' gross domestic products (GDPs), their bilateral distance, and typically an array of bilateral dummy variables assumed to reflect the bilateral trade costs between that pair of regions; we denote this the "traditional" gravity equation. This gravity equation gained acceptance among international trade economists and policymakers in the last 25 years for (at least) three reasons: formal theoretical *economic* foundations surfaced around 1980; consistently strong empirical explanatory power (high R^2 values); policy relevance for analyzing numerous free trade agreements that arose over the past 15 years.

However, the traditional gravity equation has come under scrutiny. First, the traditional specification ignores that the volume of trade from region i to region j should be influenced by trade costs between regions i and j *relative* to those of the rest-of-the-world (ROW), and the economic sizes of the ROW's regions (and prices of their goods) matter as well. Second, applications of the traditional gravity equation to study bilateral trade agreements often yielded seemingly implausible findings. For instance, coefficient estimates for dummy variables representing the effects of international economic integration agreements (EIAs) on international trade were frequently negative (cf., Frankel, 1997) and estimates of the effects of national borders (i.e.,, a national EIA) on intra-continental inter-regional trade flows were often seemingly implausibly high (cf., McCallum, 1995). The latter finding – McCallum's "border puzzle" – inspired a cottage industry of papers in the international trade literature to explain this result, cf., Anderson and Smith (1999a, 1999b) and John F. Helliwell (1996, 1997, 1998).

While two early formal theoretical foundations for the gravity equation with trade costs – first Anderson (1979) and later Bergstrand (1985) – addressed the role of "multilateral prices," a solution to the border puzzle surfaced in Anderson and van Wincoop (2003), which refined the theoretical foundations for the gravity equation to emphasize the importance of accounting properly for the endogeneity of prices.[1] Three major conclusions surfaced from the now seminal Anderson and van Wincoop (henceforth, A-vW) study, "Gravity with *Gravitas*." First, a complete derivation of a standard Armington (conditional) general equilibrium model of bilateral trade in a multi-region (N>2) setting suggests that traditional cross-section empirical gravity equations have been misspecified owing to the omission of theoretically-motivated *multilateral* (price) resistance terms for exporting and importing regions. Second, to estimate properly the full general equilibrium comparative statics of a national border or an EIA, one needs to estimate these multilateral resistance (MR) terms for any two regions with *and* without a border, in a manner

[1] Recently, Balistreri and Hillberry (2007) have questioned some aspects of Anderson and van Wincoop (2003). We address these concerns later in the paper.

consistent with theory. Third, due to the underlying nonlinearity of the structural relationships, A-vW suggest that estimation requires a custom nonlinear least squares (NLS) program to account properly for the endogeneity of prices and/or estimate the comparative static effects of a trade cost.

While the A-vW approach yields consistent, efficient estimates of gravity equation coefficients (in the absence of measurement and specification bias), Feenstra (2004, Ch. 5) notes that a "drawback" to the estimation strategy is that it requires a custom NLS program to obtain estimates. One reason the gravity equation has become the workhorse of empirical international trade in the past 25 years is that one can use ordinary linear least squares (OLS) to explain trade flows and potentially the impact of policies (such as national borders or EIAs) on such flows. Unfortunately, the need to apply custom NLS estimation has led empirical researchers typically to ignore A-vW's considerations, and will likely continue to impede incorporating these price terms into estimation of gravity equations using the A-vW approach and computation of proper comparative statics.

Another – and computationally less taxing – approach to estimate potentially unbiased gravity equation coefficients, which also acknowledges the influence of theoretically-motivated MR terms, is to use region-specific fixed effects, as noted by A-vW and Feenstra. An additional benefit is that this method avoids the measurement error associated with measuring regions' "internal distances" for the MR variables. Indeed, van Wincoop himself – and nearly every gravity equation study since A-vW – has employed this simpler technique of fixed effects, cf., Andrew Rose and Eric van Wincoop (2001) and Rose (2004). Using the case of McCallum's border puzzle as an example, Feenstra (2004, Ch. 5 Appendix) shows that fixed-effects estimation of the gravity equation can generate unbiased estimates of the *average* border effect of a pair of countries.[2]

Yet, fixed-effects estimation faces two notable drawbacks. First, without the structural system of equations, one still cannot generate region- or pair-specific comparative statics; fixed effects estimation precludes estimating MR terms with *and* without EIAs. However, empirical researchers can use fixed effects to obtain the key gravity-equation parameter estimates, and then simply construct a nonlinear system of equations to estimate multilateral price terms with and without the "border." But they don't. Outside of the A-vW Canada-U.S. trade context, researchers have not calculated the full *general-equilibrium comparative-static* effects of a free trade agreement.[3]

Second, many explanatory variables of interest are region specific; using region-specific fixed effects precludes direct estimation of partial effects of numerous potentially-important explanatory variables that are often motivated theoretically. For instance, typical gravity studies often try to estimate the effects of exporter and importer populations, foreign aid, or internal infrastructure measures on bilateral trade; such variables would be subsumed in the fixed effects, cf., Egger and Nelson (2007), Nelson and Juhasz Silva (2007), and Melitz (2007). Also, recent

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[2]In their robustness analysis, A-vW demonstrate evidence using fixed-effects for unbiased estimates of the *average* border effect. Recently, Behrens, Ertur and Koch (2007) show that OLS with fixed effects may still result in biased estimates because they fail to capture fully the spatial interdependence among trade flows and their determinants.

[3]In correspondence, Eric van Wincoop notes "people often introduce the region fixed effects to the gravity equation referring to our paper for motivation but then fail to compute (using the system of structural equations) changes in the multilateral resistance variables when doing comparative statics" (e-mail, Aug. 24, 2004).

analyses of the effects of FTAs on trade using *nonparametric* (matching) econometric techniques require indexes of multilateral resistance that are not be derived from a structural model, cf., Baier and Bergstrand (2006); an alternative approach is needed. Moreover, recent estimation of economic and political determinants of EIAs between country pairs using probit models of the likelihoods of EIAs require (exogenous) measures of multilateral resistance, cf., Mansfield and Reinhardt (2003), Baier and Bergstrand (2004), and Mansfield, Milner and Pevehouse (2008).

Consequently, the empirical researcher faces a tradeoff. A-vW's customized NLS approach can potentially generate consistent, efficient estimates of average border effects *and* comparative statics, but it is computationally burdensome relative to OLS and subject to measurement error associated with internal distance indexes. Fixed-effects estimation uses OLS and avoids internal distance measurement error for MR terms, but one cannot retrieve the multilateral price terms necessary to generate quantitative estimates of comparative-static effects without also employing the structural system of equations. Is there a third way to estimate gravity equation parameters using *exogenous* measures of multilateral resistance – *and* compute region-specific or pair-specific comparative statics – using "good old" OLS? This paper suggests a method that may be useful when NLS estimation is not suitable.

Following some background, this paper has three major parts (theory, estimation, and comparative statics). First, we provide a method for "approximating" the MR terms based upon theory. In the spirit of the recent literature on general equilibrium macroeconomic models, we use a simple first-order log-linear Taylor-series expansion of the MR terms in the A-vW system of equations to generate a reduced-form gravity equation that includes theoretically-motivated (exogenous) MR terms that can be estimated potentially using "good old" – *bonus vetus* – OLS. However – unlike fixed-effects estimation – this method can *also* generate theoretically-motivated general equilibrium comparative statics without estimating a nonlinear system of equations.[4]

Second, we discuss numerous contexts for which nonlinear estimation is not feasible and we show that our first-order log-linear-approximation method provides *virtually identical* coefficient estimates for gravity-equation parameters. For tractability, we apply our technique first to actual trade flows using the same context and Canadian-U.S. data sets as used by McCallum, A-vW, and Feenstra. However, the insights of our paper have the potential to be used in numerous contexts assessing trade-cost effects, especially estimation of the effects of tariff reductions and free trade agreements on world trade flows – the most common usage of the gravity equation in trade. We show that the linear-approximation approach works even more effectively in the context of *world* (intra- and inter-continental) trade than in the narrower context of regional (intra-continental) trade. Using Monte Carlo techniques we demonstrate that the estimated bias (of the distance elasticity) of our method over nonlinear least squares for world trade is less than *0.5 of one percent* – smaller than that for intra-continental trade flows.

Third, we demonstrate the economic conditions under which our approximation method works well to calculate comparative-static effects of key trade-cost variables . . . and when it does not. We compare the comparative statics generated using our approach versus those using A-vW's approach for the both the Canadian-U.S. context and

[4]Bergstrand, Egger, and Larch (2007) have recently shown that – in contexts outside of the Canadian-U.S. case – where bilateral trade costs are likely asymmetric solving systems of nonlinear price equations using A-vW can yield complex numbers. As noted in A-vW (2003, footnote 11), in the presence of asymmetries, "inferential identification" is "problematic."

for world trade flows with various free trade agreements using the Monte Carlo simulated data. Two important conclusions surface. First, the approximation errors (for our comparative statics) are largest when the comparative-static changes in the multilateral price terms are largest. Using simulated data from our Monte Carlo analyses, we find that the largest comparative static changes in multilateral price terms are *not necessarily* among the smallest GDP-sized economies (and consequently those with the largest trading partners). Rather, multilateral price terms change the most (for a given change in trade costs) for small countries that are *physically close*. Second, as with any linear Taylor-series expansion, approximation errors increase the further away from the center is the change, cf., Judd (1998). Since a higher-order Taylor-series expansion can reduce these errors, we discuss – based upon a second-order Taylor expansion – the factors (variances and covariances) that likely explain the approximation errors. Then, using a fixed-point iterative matrix manipulation, we show how the approximation errors can be eliminated, where the key economic insight is an NxN matrix of GDP shares *relative to bilateral distances*.

The remainder of the paper is as follows. Section 2 discusses the gravity equation literature and A-vW analysis to motivate our paper. Section 3 uses a first-order log-linear Taylor-series expansion to motivate a simple OLS regression equation that can be used to estimate average effects *and* generate comparative statics. In section 4, we apply our estimation technique to the McCallum-A-vW-Feenstra data set and compare our coefficient estimates to these papers' findings and use Monte Carlo simulations to show that estimated border effects using "good old" OLS are virtually identical to those using A-vW's technique for either *interregional* trade flows or *international* trade flows (the typical empirical context). Section 5 examines the economic conditions under which our approach approximates the comparative statics of trade-cost changes well and under which it does not. Section 6 concludes.

2. Background: The Gravity Equation and Prices

The gravity equation is now considered the empirical workhorse for studying interregional and international trade patterns, cf., Feenstra (2004). Early applications of the gravity equation – Tinbergen (1962), Linnemann (1966), Aitken (1973), and Sapir (1981) – assumed a specification similar to that used in McCallum (1995):

$$\ln \mathbf{X}_{ij} = \beta_0 + \beta_1 \ln \mathbf{GDP}_i + \beta_2 \ln \mathbf{GDP}_j - \beta_3 \ln \mathbf{DIS}_{ij} + \beta_4 \mathbf{EIA}_{ij} + \varepsilon_{ij} \qquad (1)$$

where $\mathbf{X}_{ij}$ denotes the value of the bilateral trade flow from region i to region j, $\mathbf{GDP}_i$ ($\mathbf{GDP}_j$) denotes the nominal gross domestic product of region i (j), $\mathbf{DIS}_{ij}$ denotes the distance (typically in miles or nautical miles) from the economic center of region i to that of region j, and $\mathbf{EIA}_{ij}$ is a dummy variable assuming the value 1 (0) if two regions share (do not share) an economic integration agreement. In the McCallum Canada-U.S. context, $\mathbf{EIA}_{ij}$ would be a national "border" dummy reflecting membership in the same country. In the remainder of this paper, boldfaced regular-case (non-bold italicized) variable names denote observed (unobserved) variables. Traditionally, economists have focused on estimates of, say, β_4, to measure the "average" (treatment) effect of an EIA on trade from i to j. Traditional specification (1) typically excludes *price* terms. The rationale in the studies was that prices were endogenous and consequently would not surface in the reduced-form cross-section bilateral trade flow equation.[5]

However, theoretical foundations in Anderson (1979), Bergstrand (1985), Deardorff (1998), Eaton and Kortum (2002), A-vW (2003), and Feenstra (2004) all suggest that traditional gravity equation (1) is likely misspecified owing to the omission of measures of multilateral resistance (or prices). In reality, the trade flow from i to j is surely influenced by the prices of products in the other N–2 regions in the world, which themselves are influenced by the bilateral distances (and EIAs, etc.) of each of i and j with the other N–2 regions. Bergstrand (1985) provided early empirical evidence of this omitted variables bias, but was limited by crude price-index data. As Feenstra (2004) reminds us, published price indexes probably do not reflect accurately "true" border costs (numerous costs associated with international transactions) and are measured relative to an arbitrary base period.

A-vW raised two important considerations. First, A-vW showed theoretically that proper estimation of the coefficients of a theoretically-based gravity equation needs to account for the influence of endogenous price terms. Second, estimation yields partial effects of a change in a bilateral trade cost on a bilateral trade flow, but not *general-equilibrium* effects. A-vW clarified that the comparative-static effects of a change in a trade cost were influenced by the full general-equilibrium framework.

2.1. The A-vW Theoretical Model

To understand the context, we initially describe a set of assumptions to derive a gravity equation; for analytical details, see A-vW (2003). First, assume a world endowment economy with N regions and N (aggregate) goods, each good differentiated by origin. Second, assume consumers in each region j have identical constant-elasticity-of-substitution (CES) preferences:

$$U_{\mathrm{j}} = \left[\sum_{\mathrm{i}=1}^{\mathrm{N}} C_{\mathrm{ij}}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \qquad \mathrm{j} = 1,\ldots,\mathrm{N} \tag{2}$$

where U_{j} is the utility of consumers in region j, C_{ij} is consumption of region i's good in region j, and σ is the elasticity of substitution (assuming $\sigma > 1$).[6] Maximizing (2) subject to the budget constraint:

$$Y_{\mathrm{j}} = \sum_{\mathrm{i}=1}^{\mathrm{N}} p_{\mathrm{i}} t_{\mathrm{ij}} C_{\mathrm{ij}} \tag{3}$$

[5]The traditional argument is as follows. Suppose importer j's demand for the trade flow from i to j is a function of j's GDP, the price of the product in i (p_{i}), and distance from i to j. Suppose exporter i's supply of goods is a function of i's GDP and p_{i}. Market clearing would require county i's export supply to equal the sum of the N-1 bilateral import demands (in an N-country world). This generates a system of N+1 equations in N+1 endogenous variables: N-1 bilateral import demands $X_{\mathrm{ij}}^{\mathrm{D}}$ (j = 1,...,N with j≠i), supply variable $X_{\mathrm{i}}^{\mathrm{S}}$, and price variable p_{i}. This system could be solved for a bilateral trade flow equation for $\mathbf{X}_{\mathrm{ij}}$ that is a function of the GDPs of i and j and their bilateral distance. Then p_{i} is endogenous and excluded from the reduced-form bilateral trade flow gravity equation.

[6]Consumption is measured as a quantity. We can also set up the model in terms of a representative consumer with M_{j} consumers in each country, but the results are analytically identical.

where p_i is the exporter's price of region i's good and t_{ij} is the gross trade cost (one plus the *ad valorem* trade cost[7]) associated with exports from i to j, yields a set of first-order conditions that can be solved for the demand for the nominal bilateral trade flow from i to j (X_{ij}):

$$X_{ij} = \left(\frac{p_i t_{ij}}{P_j} \right)^{1-\sigma} Y_j \quad (4)$$

where $X_{ij} = p_i\, t_{ij}\, C_{ij}$ and P_j is the CES price index, given by:

$$P_j = \left[\sum_{i=1}^{N} (p_i t_{ij})^{1-\sigma} \right]^{1/(1-\sigma)} \quad (5)$$

Third, an assumption of market clearing requires:

$$Y_i = \sum_{j=1}^{N} X_{ij} \quad (6)$$

Following A-vW, substitution of (4) and (5) into (6) and some algebraic manipulation yields:

$$X_{ij} = \left(\frac{Y_i Y_j}{Y^T} \right) \left(\frac{t_{ij}}{P_i P_j} \right)^{1-\sigma} \quad (7)$$

where it follows that

$$P_i = \left[\sum_{j=1}^{N} \left(\theta_j / t_{ij}^{\sigma-1} \right) P_j^{\sigma-1} \right]^{1/(1-\sigma)} \quad (8)$$

$$P_j = \left[\sum_{i=1}^{N} \left(\theta_i / t_{ij}^{\sigma-1} \right) P_i^{\sigma-1} \right]^{1/(1-\sigma)} \quad (9)$$

under a fourth assumption that bilateral trade barriers t_{ij} and t_{ji} are equal for all pairs. In equations (8) and (9), Y^T denotes total income of all regions, which is constant across region pairs and θ_i (θ_j) denotes Y_i / Y^T (Y_j / Y^T). It will be useful now to define the term "economic density." For country i, the bilateral "economic density" of a trading partner j is the amount of economic activity in j relative to the cost of trade between i and j (scaled by σ-1 > 0), or $\theta_j / t_{ij}^{\sigma-1}$.

2.2. The Econometric Model

[7] As conventional, we assume that all trade costs consume resources and can be interpreted as goods "lost in transit."

As is common to this literature, for an econometric model we assume the log of the observed trade flow ($\ln\mathbf{X}_{ij}$) is equal to the log of the true trade flow ($\ln X_{ij}$) plus a log-normally distributed error term (ε_{ij}). Y_i can feasibly be represented empirically by observable $\mathbf{GDP}_i$. However, the world is not so generous as to provide observable measures of bilateral trade costs t_{ij}. Following the literature, a fifth assumption is that the gross trade cost factor is a log-linear function of *observable* variables, such as bilateral distance ($\mathbf{DIS}_{ij}$) and $e^{-\alpha\mathbf{EIA}_{ij}}$, the latter representing the *ad valorem* equivalent of a common EIA, respectively:

$$t_{ij} = \mathbf{DIS}_{ij}^{\rho} e^{-\alpha\mathbf{EIA}_{ij}} \tag{10}$$

where $e^{-\alpha\mathbf{EIA}_{ij}}$ equals $e^{-\alpha}$ (< 1) if the two regions are in an economic integration agreement (assuming $\alpha > 0$). One could also include a language dummy, an adjacency dummy, etc.; for brevity, we ignore these.

In the McCallum-AvW-Feenstra context of Canadian provinces and U.S. states, $\mathbf{EIA}_{ij} = 1$if the two regions are in the same country, and 0 otherwise. In the context of the theory, estimation of the gravity equation's parameters should account for the MR terms defined in equations (8) and (9). A-vW describe one customized nonlinear procedure for estimating equations (7)-(10) to generate unbiased estimates in a two-country world with 10 Canadian provinces, 30 U.S. states and an aggregate rest-of-U.S. (the other 20 states plus the District of Columbia), or 41 regions total. A-vW also estimate a multicounty model; discussion of that is treated later. This procedure requires minimizing the sum-of-squared residuals of:

$$\begin{aligned}\ln\left[\mathbf{X}_{ij} / \left(\mathbf{GDP}_i \mathbf{GDP}_j\right)\right] = a_{\mathbf{0}} + a_1 \ln\mathbf{DIS}_{ij} + a_2\mathbf{EIA}_{ij} \\ - \ln P_i^{1-\sigma} - \ln P_j^{1-\sigma} + \varepsilon_{ij}\end{aligned} \tag{11}$$

subject to the 41 market-equilibrium conditions (j = 1,...,41):

$$P_j^{1-\sigma} = \sum_{k=1}^{41} P_k^{\sigma-1}\left(\mathbf{GDP}_k / \mathbf{GDP}^{\mathbf{T}}\right) e^{a_1 \ln\mathbf{DIS}_{kj} + a_2\mathbf{EIA}_{kj}} \tag{12}$$

to estimate a_0, a_1, and a_2 where, in the model's context, $a_0 = -\ln\mathbf{GDP}^{\mathbf{T}}$, $a_1 = -\rho\,(\sigma\text{-}1)$ and $a_2 = -\alpha\,(\sigma\text{-}1)$. This obviously requires a custom NLS program.

2.3. Estimating Comparative-Static Effects

As A-vW stress, the MR terms $P_i^{1-\sigma}$ and $P_j^{1-\sigma}$ are "critical" to understanding the impact of border barriers on bilateral trade. Once estimates of a_0, a_1, and a_2 are obtained, one can then retrieve estimates of $P_i^{1-\sigma}$ and $P_j^{1-\sigma}$ for all j = 1,...,41 regions both in the presence and absence of a national border. Let $\mathbf{P}_i^{1-\sigma}$ ($\mathbf{P}^{*}{}_i^{1-\sigma}$) denote the estimate of the MR region *i* with (without) an EIA following NLS estimation of equations (11) and (12). In the context of the model, A-vW and Feenstra (2004) both show that the ratio of bilateral trade between any two regions *with* an EIA ($\mathbf{X}_{ij}$) and *without* an EIA ($\mathbf{X}^{*}_{ij}$) is given by:

$$\mathbf{X}_{ij} / \mathbf{X}^*_{ij} = e^{a_2 \mathbf{EIA}_{ij}} \left(\mathbf{P}_i^{*1-\sigma} / \mathbf{P}_i^{1-\sigma} \right) \left(\mathbf{P}_j^{*1-\sigma} / \mathbf{P}_j^{1-\sigma} \right) \tag{13}$$

Comparative-static effects of an integration agreement are then calculated using equation (13).

Consequently, A-vW (2003) "resolved" the border puzzle theoretically and empirically. However, the appealing characteristic of the gravity equation, that likely has contributed to its becoming the workhorse for the study of empirical trade patterns, is that it has been estimated for decades using OLS. The A-vW procedure cannot use OLS, which will likely inhibit future researchers from recognizing empirically the MR terms. Moreover, in some instances mentioned earlier and later, one might want to have exogenous measures of the MR terms motivated by theory.

A-vW (2003) and Feenstra (2004) both note that a ready alternative to estimating consistently the *average* border effect is to apply fixed effects. However, while fixed effects can determine gravity equation parameters consistently, estimation of country-specific border effects *still requires* construction of the structural system of price equations to distinguish MR terms with *and* without borders. We demonstrate in this paper a simple technique that yields virtually identical estimates of the average effects *and* (in many instances) comparative statics surfaces by applying a Taylor-series expansion to the theory.

3. Theory: Simplifying Anderson and van Wincoop's "Simplified" Gravity Equation

In this section, we apply a first-order log-linear Taylor-series expansion to the system of price equations above to generate a reduced-form gravity equation – including theoretically-motivated exogenous multilateral-and-world-resistance (MWR) terms – that can be estimated using OLS, and will be used later in section 4. The key methodological insight is the use of a first-order Taylor-series expansion, not commonly used in international trade but the *workhorse* for modern dynamic macroeconomics. A first-order Taylor-series expansion of any function $f(x_i)$, centered at x, is given by $f(x_i) = f(x) + [f'(x)](x_i - x)$. In modern dynamic macroeconomics, the expansion is usually made around the steady-state value suggested by the underlying theoretical model.[8]

Since the solution to a Taylor-series expansion is sensitive to how it is centered, we use in our static trade context the natural choice of an expansion centered around a world with symmetric trade frictions – $t_{ij} = t$ – but allowing asymmetric economic sizes.[9] We begin with N equations (8) from Section 2. Dividing both sides of equation (8) by $t^{1/2}$ yields:

$$P_i / t^{1/2} = \left[\sum_{j=1}^{N} \theta_j \left(t_{ij} / t^{1/2} \right)^{1-\sigma} / P_j^{1-\sigma} \right]^{1/(1-\sigma)} = \left[\sum_{j=1}^{N} \theta_j \left(t_{ij} / t \right)^{1-\sigma} / \left(P_j / t^{1/2} \right)^{1-\sigma} \right]^{1/(1-\sigma)} \tag{14}$$

Define $\tilde{P}_i = P_i / t^{1/2}$, $\tilde{P}_j = P_j / t^{1/2}$ and $\tilde{t}_{ij} = t_{ij} / t$. Substituting these expressions into equation (14) yields:

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[8] We find using a Monte Carlo robustness analysis that a first-order Taylor series works well for estimating gravity equation coefficients. Higher-order terms are largely unnecessary for estimation. However, such terms are relevant for subsequent comparative statics; we address this more later.

[9] We are grateful to a referee that suggested this center that collapsed two cases into one more general case.

$$\tilde{P}_{\mathrm{i}} = \left[\sum_{\mathrm{j=1}}^{\mathrm{N}} \theta_{\mathrm{j}} \left(\tilde{t}_{\mathrm{ij}} / \tilde{P}_{\mathrm{j}} \right)^{1-\sigma} \right]^{1/(1-\sigma)} \tag{15}$$

for i = 1,..., N, where $\tilde{P}_{\mathrm{i}} = P_{\mathrm{i}} / t^{1/2}$, $\tilde{P}_{\mathrm{j}} = P_{\mathrm{j}} / t^{1/2}$ and $\tilde{t}_{\mathrm{ij}} = t_{\mathrm{ij}} / t$. It will be useful for later to rewrite (15) as:

$$e^{(1-\sigma)\ln \tilde{P}_{\mathrm{i}}} = \sum_{\mathrm{j=1}}^{\mathrm{N}} e^{\ln \theta_{\mathrm{j}}} e^{(\sigma-1)\ln \tilde{P}_{\mathrm{j}}} e^{(1-\sigma)\ln \tilde{t}_{\mathrm{ij}}} \tag{16}$$

where e is the natural logarithm operator.

In a world with symmetric trade costs ($t > 0$), $t_{\mathrm{ij}} = t$, implying $\tilde{t}_{\mathrm{ij}} = 1$. In this world, the latter implies:

$$\tilde{P}_{\mathrm{i}}^{1-\sigma} = \sum_{\mathrm{j=1}}^{\mathrm{N}} \theta_{\mathrm{j}} \tilde{P}_{\mathrm{j}}^{\sigma-1} \tag{17}$$

for all i =1,..., N. Multiplying both sides of equation (17) by $\tilde{P}_{\mathrm{i}}^{\sigma-1}$ yields:

$$1 = \sum_{\mathrm{j=1}}^{\mathrm{N}} \theta_{\mathrm{j}} \left(\tilde{P}_{\mathrm{i}} \tilde{P}_{\mathrm{j}} \right)^{\sigma-1} \tag{18}$$

As noted in Feenstra (2004, p. 158, footnote 11), the solution to equation (18) is:

$$\tilde{P}_{\mathrm{i}} = \tilde{P}_{\mathrm{j}} = \tilde{P} = 1 \tag{19}$$

Hence, under symmetric trade costs ($t_{\mathrm{ij}} = t$), $\tilde{t}_{\mathrm{ij}} = \tilde{P}_{\mathrm{i}} = \tilde{P}_{\mathrm{j}} = 1$ and it follows that $P_{\mathrm{i}} = P_{\mathrm{j}} = t^{1/2}$.

A first-order log-linear Taylor-series expansion of equation (16) centered at $\tilde{t} = \tilde{P} = 1$ (and $\ln \tilde{t} = \ln \tilde{P} = 0$) is:

$$1 + \ln \tilde{P}_{\mathrm{i}}^{1-\sigma} = 1 - \sum_{\mathrm{j=1}}^{\mathrm{N}} \theta_{\mathrm{j}} \ln \tilde{P}_{\mathrm{j}}^{1-\sigma} + (1-\sigma) \sum_{\mathrm{j=1}}^{\mathrm{N}} \theta_{\mathrm{j}} \ln \tilde{t}_{\mathrm{ij}} \tag{20}$$

using $d\left[e^{(1-\sigma)\ln \tilde{P}} \right] / d(\ln \tilde{P}) = (1-\sigma) e^{(1-\sigma)\ln \tilde{P}}$. Subtracting 1 from both sides, multiplying both sides by θ_{i},

and summing both sides over N yields:

$$\sum_{\mathrm{i=1}}^{\mathrm{N}} \theta_{\mathrm{i}} \ln \tilde{P}_{\mathrm{i}}^{1-\sigma} = -\sum_{\mathrm{i=1}}^{\mathrm{N}} \theta_{\mathrm{i}} \sum_{\mathrm{j=1}}^{\mathrm{N}} \theta_{\mathrm{j}} \ln \tilde{P}_{\mathrm{j}}^{1-\sigma} + (1-\sigma) \sum_{\mathrm{i=1}}^{\mathrm{N}} \sum_{\mathrm{j=1}}^{\mathrm{N}} \theta_{\mathrm{i}} \theta_{\mathrm{j}} \ln \tilde{t}_{\mathrm{ij}} \tag{21}$$

Noting that the first RHS term can be expressed in alternative ways,

$$-\sum_{i=1}^{N}\theta_i \sum_{j=1}^{N}\theta_j \ln \tilde{P}_j^{1-\sigma} = -\sum_{j=1}^{N}\theta_j \ln \tilde{P}_j^{1-\sigma} = -\sum_{i=1}^{N}\theta_i \ln \tilde{P}_i^{1-\sigma}$$

we can substitute $-\sum_{i=1}^{N}\theta_i \ln \tilde{P}_i^{1-\sigma}$ for $-\sum_{i=1}^{N}\theta_i \sum_{j=1}^{N}\theta_j \ln \tilde{P}_j^{1-\sigma}$ in equation (21) to yield:

$$\sum_{i=1}^{N}\theta_i \ln \tilde{P}_i^{1-\sigma} = -\sum_{j=1}^{N}\theta_i \ln \tilde{P}_i^{1-\sigma} + (1-\sigma)\sum_{i=1}^{N}\sum_{j=1}^{N}\theta_i\theta_j \ln \tilde{t}_{ij}$$

or

$$\sum_{i=1}^{N}\theta_i \ln \tilde{P}_i^{1-\sigma} = \sum_{j=1}^{N}\theta_j \ln \tilde{P}_j^{1-\sigma} = (1/2)(1-\sigma)\sum_{i=1}^{N}\sum_{j=1}^{N}\theta_i\theta_j \ln \tilde{t}_{ij} \tag{22}$$

Substituting equation (22) into equation (20), after subtracting 1 from both sides of eq. (20), yields::

$$\ln \tilde{P}_i^{\sigma-1} = -\ln \tilde{P}_i^{1-\sigma} = (\sigma-1)\left[\sum_{j=1}^{N}\theta_j \ln \tilde{t}_{ij} - (1/2)\sum_{i=1}^{N}\sum_{j=1}^{N}\theta_i\theta_j \ln \tilde{t}_{ij}\right] \tag{23}$$

Recalling that $\ln\tilde{P}_i^{\sigma-1} = (\sigma-1)\ln P_i - ½(\sigma-1)\ln t$ and $\ln \tilde{t}_{ij} = \ln t_{ij} - \ln t$, then substitution into the equation above and some algebraic manipulation yields:

$$\ln P_i^{\sigma-1} = -\ln P_i^{1-\sigma} = (\sigma-1)\left[\sum_{j=1}^{N}\theta_j \ln t_{ij} - (1/2)\sum_{i=1}^{N}\sum_{j=1}^{N}\theta_i\theta_j \ln t_{ij}\right] \tag{24}$$

and it follows that:

$$\ln P_j^{\sigma-1} = -\ln P_j^{1-\sigma} = (\sigma-1)\left[\sum_{i=1}^{N}\theta_i \ln t_{ji} - (1/2)\sum_{i=1}^{N}\sum_{j=1}^{N}\theta_i\theta_j \ln t_{ij}\right] \tag{25}$$

Although (by assumption) $t_{ij} = t_{ji}$, $\sum_{i=1}^{N}\theta_i \ln t_{ij}$ need not equal $\sum_{j=1}^{N}\theta_j \ln t_{ij}$.[10]

Equations (24) and (25) are critical to understanding this analysis. The benefit of the first-order log-linear expansion is that it identifies the *exogenous* actual "multilateral resistance" factors determining the multilateral price terms in equations (7)-(9) in a manner consistent with the theoretical model. To understand the intuition behind equation (25) – analogous for (24) – we consider separately each of the two components of the RHS. The first component is a GDP-share-weighted (geometric) average of the gross trade costs facing country j across all regions. The higher this average, the greater overall multilateral resistance in j. Holding constant bilateral determinants of trade, the larger is j's multilateral resistance, the lower are bilateral trade costs relative to multilateral trade costs.

[10]For instance, internal distances t_{ii} and t_{jj} will likely differ, as will θ_i and θ_j. For transparency and consistency with A-vW's notation, we note that $\ln P_i^{\sigma-1} = -\ln P_i^{1-\sigma}$; analogously for j.

Hence, the *larger* the bilateral trade flow from i to j will be.

Now consider the second component on the RHS of equation (25). The Taylor-series expansion here makes more transparent the influence of *world resistance*, which is identical for all countries. In A-vW, this second component was also present, cf., A-vW's equations (14)-(16), but not emphasized. World resistance lowers trade between *every* pair of countries. This term is constant in cross-section gravity estimation, embedded in and affecting only the intercept. (However, the term cannot be ignored in estimating "border effects.")[11] Together, these terms indicate that the level of bilateral trade from i to j is influenced – not just by the level of *bilateral relative to multilateral* trade costs, but also – by *multilateral relative to world* trade costs.

In the context of the theory just discussed, we can obtain consistent estimates of the gravity equations' coefficients – accounting for the endogenous multilateral price variables – by estimating *using OLS* the reduced-form gravity equation:

$$\ln \mathbf{X}_{ij} = \beta_0' + \ln \mathbf{GDP}_i + \ln \mathbf{GDP}_j - (\sigma - 1)\ln t_{ij}$$
$$+ (\sigma - 1)\left[\left(\sum_{j=1}^{N} \theta_j \ln t_{ij}\right) - \frac{1}{2}\left(\sum_{i=1}^{N}\sum_{j=1}^{N} \theta_i \theta_j \ln t_{ij}\right)\right] \tag{26}$$
$$+ (\sigma - 1)\left[\left(\sum_{i=1}^{N} \theta_i \ln t_{ji}\right) - \frac{1}{2}\left(\sum_{i=1}^{N}\sum_{j=1}^{N} \theta_i \theta_j \ln t_{ij}\right)\right]$$

where $\beta_0' = -\ln Y^T$ is a constant across country pairs, as is $\sum_{i=1}^{N}\sum_{j=1}^{N} \theta_i \theta_j \ln t_{ij}$. Thus, in the context of the theoretical model, the influence of the endogenous multilateral price variables can be accounted for – once we have measures of t_{ij} – using these theoretically-motivated *exogenous* multilateral resistance variables.

We close this section noting that it is useful to exponentiate equation (26). After some algebra, this yields:

$$\frac{X_{ij}}{Y_i Y_j / Y^T} = \left(\frac{t_{ij}}{t_i(\theta)\ t_j(\theta) / t^T(\theta)}\right)^{-(\sigma-1)} \tag{27}$$

where $t_i(\theta) = \Pi_{j=1}^{N} t_{ij}^{\theta_j}$, $t_j(\theta) = \Pi_{i=1}^{N} t_{ji}^{\theta_i}$, $t^T(\theta) = \Pi_{i=1}^{N}\Pi_{j=1}^{N} t_{ij}^{\theta_i\theta_j}$, and recall $\theta_i = Y_i/Y^T$ and $t_{ij} = t_{ji}$ (by assumption). Our use of the Taylor-series expansion simplifies further the "significantly simplified" gravity equation implied by A-vW's equations (7)-(9), cf., A-vW (2003, p. 176). Equation (27) is a simple reduced-form equation capturing the

[11] Moreover, in panel estimation, changes in world resistance over time – along with changes in world income – provide a rationale for including a time trend.

theoretical influences of bilateral, multilateral, and world trade costs on (relative) bilateral trade. As noted, multilateral-and-world-trade costs are GDP-share weighted. Given data on bilateral trade flows, national incomes, and bilateral trade costs, equation (26) can be estimated by "good old" OLS – noting the possible *endogeneity bias* introduced by GDP-share weights in RHS variables.[12] But will this equation work *empirically*? Moreover, even if our approach yields consistent estimates of gravity-equation parameters, can our approach provide "good" approximations of the MR terms *and* the comparative statics generated using A-vW's nonlinear approach? The next two sections address these two questions in turn.

4. Estimation: *Bonus Vetus* OLS

The goal of this section is to show that one can generate virtually identical gravity equation coefficient estimates ("partial" effects) to those generated using the technique in A-vW but using instead OLS with exogenous multilateral-resistance terms suggested in the previous section. While the approach should work in numerous contexts, for tractability we apply it first in section A to McCallum's U.S.-Canadian case, since this is a popular context. We estimate the McCallum, A-vW, fixed-effects, and our versions of the model using the A-vW data provided at Robert Feenstra's website, and compare our coefficient estimates with the other results. We show that A-vW, fixed effects, and our methods can yield similar gravity-equation coefficient estimates, even though both BV-OLS and fixed effects are computationally simpler. In section B, we provide Monte Carlo analyses for two contexts: Canadian-U.S. flows *and* world trade flows among 88 countries. In section C, we discuss three contexts in which our method would be useful for estimating gravity-equation parameters instead of using fixed effects.

Before implementing equation (26) econometrically, three issues need to be addressed. First, in implementing theoretically the Taylor-series expansion, we needed to assume a "center" for the expansion. In the theory (in part, at one referee's suggestion) we centered the expansion around a symmetric trade cost, t. However, OLS generates estimates of coefficients based upon covariances and variances of variables around *all* variables' "means." Consequently, for estimation purposes – but *not* for comparative-static exercises later – a more useful center would be an expansion around a symmetric world, that is, a world symmetric in all variables (trade costs *and* economic sizes). In such a world, the first-order log-linear Taylor expansion of the same system of multilateral price equations yields a reduced-form analogue to equation (26) that simply replaces the GDP-share weights (θ_i, θ_j) with equal weights (1/N); derivations are available on request.

Second, even if one wanted to generate the econometric specification suggested strictly by theory, another econometric issue arises. As in A-vW, in the estimation trade flows were scaled by the product of GDPs to impose unitary income elasticities and also to avoid an endogeneity bias (running from trade flows to GDPs). Including GDP-share-weighted multilateral trade costs could create an endogeneity bias. Hence, for both reasons mentioned,

[12] We ignore here the possibility of "zero" trade flows. Such issues have been dealt with by various means; see, for example, Felbermayr and Kohler (2004).

we use the simple averages of the trade costs for estimation.[13]

Third, to implement equation (26) empirically we need to replace the unobservable theoretical trade-cost variable t_{ij} in (26) with an *observable* variable. Following the literature, equation (10) earlier suggests two typical observable variables likely influencing unobservable t_{ij} – bilateral distance ($\mathbf{DIS}_{ij}$) and a dummy representing the presence of absence of an economic integration agreement ($\mathbf{EIA}_{ij}$). We define a dummy variable, $\mathbf{BORDER_{ij}}$, which assumes a value of 1 if regions i and j are *not* in the same nation; hence, $\mathbf{EIA_{ij}} = 1 - \mathbf{BORDER_{ij}}$.[14] Taking the logarithms of both sides of equation (10) and then substituting the resulting equation for ln t_{ij} into (26) – and using equal weights (1/N) rather than GDP-share weights (θ_i) – yields:

$$\ln \mathbf{x}_{ij} = \beta_0' - \rho(\sigma-1)\ln \mathbf{DIS}_{ij} - \alpha(\sigma-1)\mathbf{BORDER}_{ij} + \rho(\sigma-1)\mathbf{MWRDIS}_{ij} + \alpha(\sigma-1)\mathbf{MWRBORDER}_{ij} + \varepsilon_{ij} \quad (28)$$

where

$$\mathbf{MWRDIS}_{ij} = \left[\frac{1}{N}\left(\sum_{j=1}^{N}\ln \mathbf{DIS}_{ij}\right) + \frac{1}{N}\left(\sum_{i=1}^{N}\ln \mathbf{DIS}_{ij}\right) - \frac{1}{N^2}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\ln \mathbf{DIS}_{ij}\right)\right] \text{ and} \quad (29)$$

$$\mathbf{MWRBORDER}_{ij} = \left[\frac{1}{N}\left(\sum_{j=1}^{N}\mathbf{BORDER}_{ij}\right) + \frac{1}{N}\left(\sum_{i=1}^{N}\mathbf{BORDER}_{ij}\right) - \frac{1}{N^2}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\mathbf{BORDER}_{ij}\right)\right] \quad (30)$$

where $\mathbf{x}_{ij} = \mathbf{X}_{ij} / \mathbf{GDP}_i\, \mathbf{GDP}_j$. To conform to our theory, coefficient estimates for ln**DIS** (**BORDER**) and **MWRDIS** (**MWRBORDER**) are restricted to have identical but oppositely-signed coefficient values. "MWR" denotes Multilateral and World Resistance. As discussed above, to conform to OLS, only estimation of (28) uses equally-weighted trade-cost variables; comparative statics will use θ-weighted trade costs.

As readily apparent, equation (28) can be estimated using OLS, once data on trade flows, GDPs, bilateral distances, and borders are provided. We note that the inclusion of these additional MWR terms appears reminiscent of early attempts to include – what A-vW term – "atheoretical remoteness" variables, typically GDP-weighted averages of each country's distance from all of its trading partners. However, there are two important differences here. First, our additional (the last two) terms are motivated by theory; moreover, we make explicit the role of *world*

[13] Monte Carlo analyses confirm that estimates are marginally less biased using the simple averages of RHS variables, rather than the GDP-weighted averages. However, GDP-share-weighted MR terms will generate less biased predicted values of comparative statics. These results are confirmed in Bergstrand, Egger, and Larch (2007).

[14] It will be useful now to distinguish "regions" from "countries." We assume that a country is composed of regions (which, for empirical purposes later, can be considered states or provinces). We will assume N regions in the world and n countries, with N > n. Our theoretical model applies to a two-country or multi-country (n > 2) world. We will assume n ≥ 2. A "border" separates countries. Also, we use **BORDER** rather than **EIA** so that the coefficient estimates for **DIS** and **BORDER** are both negative and therefore are consistent with A-vW (2003) and Feenstra (2004). The model is isomorphic to being recast in a monopolistically-competitive framework.

resistance. Second, previous atheoretical remoteness measures included only multilateral *distance*, ignoring all other multilateral (and world) "border" variables (such as adjacency, language, etc.).

4.1. Estimation using the McCallum-A-vW-Feenstra Data Set for Actual Canadian-U.S. Trade Flows

We follow the A-vW procedure (for the two-country model) of estimating the gravity equation for trade flows among 10 Canadian provinces, 30 U.S. states, and one aggregate region representing the other 20 U.S. states and the District of Columbia (denoted RUS). As in A-vW, we do not include trade flows internal to a state or province. We calculate the distance between the aggregate U.S. region and the other regions in the same manner as A-vW. We also compute and use the *internal distances* as described in A-vW for **MWRDIS**. Hence, there are 41 regions. Some trade flows are zero and, as in A-vW, these are omitted. As in A-vW and Feenstra (2004), we have 1511 observations for trade flows from year 1993.

Table 1 provides the results. For purposes of comparison, column (1) of Table 1 provides the benchmark model (McCallum) results estimating equation (28) except *omitting* **MWRDIS** and **MWRBORDER**. Columns (2) and (3) provide the model estimated using NLS as in A-vW for the two-country and multi-country cases, respectively. Column (4) provides the results from estimating equation (28). For completeness, column (5) provides the results from estimating equation (28), but using region-specific fixed effects instead of **MWRDIS** and **MWRBORDER**.

Table 1's results are generally comparable to Table 2 in A-vW. Column (1)'s coefficient estimates for the basic McCallum regression, ignoring multilateral resistance terms, are biased, as expected. This specification can be compared with Feenstra (2004, Table 5.2, column 3), since it uses US-US, CA-CA, and US-CA data for 1993. Note, however, we report the border dummy's coefficient estimate ("Indicator border") whereas Feenstra reports instead the implied "Country Indicator" estimates.[15] Columns (2) and (3) in Table 1 report the estimates (using *GAUSS*) of the A-vW benchmark coefficient estimates; these correspond exactly to those in A-vW's Table 2 and (for the two-country case) Feenstra's Table 5.2, column (4). The coefficient estimates from our OLS specification (28) are reported in column (4) of Table 1. While our coefficient estimates differ from the NLS estimates in columns (2) and (3), they match closely the coefficient estimates using fixed effects in column (5). Recall that – as both A-vW and Feenstra note – fixed effects should provide unbiased coefficient estimates of the bilateral distance and bilateral border effects, accounting fully for multilateral-resistance influences in estimation. Our column (5) estimates match exactly those in A-vW and Feenstra (2004).[16]

We now address the difference between bilateral distance coefficient estimates in columns (2) and (3) and

[15] In Feenstra's Table 5.2, column 3, he does not report the actual dummy variable's coefficient estimate (comparable to our estimate of 0.71). Instead, he reports only the implied "Indicator Canada" and "Indicator US" estimates of 2.75 and 0.40, respectively. The implied Indicator Canada and Indicator US estimates from our regression are 2.66 and 0.48, respectively; the difference is that we restrict the GDP elasticities to unity. When we relax the constraints on GDP elasticities, our estimates match those in Feenstra's Table 5.2, column 3 and A-vW's Table 1 exactly.

[16] The coefficient estimates from the fixed-effects regression in A-vW's Table 6, column (viii) are not reported. However, they were generously provided by Eric van Wincoop in e-mail correspondence, along with the other coefficient estimates associated with their Table 6. A-vW's Distance (Border) coefficient estimate using fixed effects was -1.25 (-1.54).

those in columns (4) and (5). While Feenstra (2004) omitted addressing this difference, A-vW did address it in their sensitivity analysis (2003, part V, Table 6). As A-vW (2003, p. 188) note, the bilateral distance coefficient estimate using their NLS program is quite sensitive to the calculation of "internal distances." In their sensitivity analysis, they provide alternative coefficient estimates when the internal distance variable values are doubled (or, 0.5 minimum capitals' distance). These are reported in column (6) of our Table 1; note that the absolute value of the distance coefficient increases with virtually no change in the border dummy's coefficient estimate. Using the same procedure, we increased the internal distance variables' values by a factor of ten (or, 2.5 times minimum capitals' distance); we see in column (7) that the bilateral distance coefficient estimate is now much closer to those in columns (4) and (5).

These results confirm A-vW's suspicion that the NLLS estimation technique is sensitive to both measurement error in internal distances and potential specification error. The main reason is the interaction of the distance and border-dummy variables using NLS. Fixed-effects estimates, of course, do not depend on internal distance measures. Our OLS estimation procedure avoids the potential bias introduced by measurement error and potential specification error better than the nonlinear estimation procedure. First, our OLS estimates are insensitive to measures of internal distance. As A-vW note (p. 179), internal distances are only relevant to calculating the multilateral resistance terms (in our context, only the multilateral and world resistance (MWR) terms). Examine equation (29) closely. Since **MWRDIS** is linear in logs of distance, a doubling of internal distance simply alters the intercept of equation (28). For instance, we can rewrite $\mathbf{MWRDIS}_{ij}$ as a function of the internal distance measures ($\ln \mathbf{DIS}_{ii}$ for all i=1,...,N) and all other bilateral distances ($\ln \mathbf{DIS}_{ij}$ for all $i \neq j$), which we denote $\mathbf{Other}_{ij}$:

$$\mathbf{MWRDIS}_{ij} = \left[\frac{\ln \mathbf{DIS}_{ii}}{N} + \frac{\ln \mathbf{DIS}_{jj}}{N} - \frac{\ln \mathbf{DIS}_{11}}{N^2} - \ldots - \frac{\ln \mathbf{DIS}_{NN}}{N^2} + \mathbf{Other}_{ij} \right] \quad (31)$$

Now double all internal distances in equation (31). This yields:

$$\mathbf{MWRDIS}_{ij} = \left[\frac{\ln \mathbf{DIS}_{ii}}{N} + \frac{\ln 2}{N} + \frac{\ln \mathbf{DIS}_{jj}}{N} + \frac{\ln 2}{N} - \frac{\ln \mathbf{DIS}_{11}}{N^2} - \ldots - \frac{\ln \mathbf{DIS}_{NN}}{N^2} - \frac{N \ln 2}{N^2} + \mathbf{Other}_{ij} \right] \quad (32)$$

This alters $\mathbf{MWRDIS}_{ij}$ by a constant, (ln 2)/N, for all pairs (i,j). This simply scales $\mathbf{MWRDIS}_{ij}$ by a constant, and thus will have *no effect* on the coefficient estimates of equation (28); measurement error introduced by internal distances in A-vW's structural estimation is avoided using fixed effects or our OLS estimation. Second, OLS avoids potential specification bias, such as one raised by Balistreri and Hillberry (2004) noting A-vW's estimates ignored the constraint that the constant (a_0) needed to equal (the negative of the log of) world income; once this structural constraint is imposed, the A-vW coefficient estimates (especially that for distance) are closer to the fixed-effects estimates and our estimates (results available on request). OLS and fixed effects avoid this specification error.[17]

4.2. Monte Carlo Analyses

The previous section addressed the question: Does OLS estimation with exogenous MR terms work empirically as an approximation to A-vW (allowing for measurement and specification error)? While NLS estimation of the A-vW system of equations, our OLS specification, and a fixed-effects specification should all generate similar estimates of $-\rho(\sigma\text{-}1)$ and $-\alpha(\sigma\text{-}1)$, a comparison of Table 1's empirical results for specifications (2)-(5) yield significantly different results. Notably, our OLS (spec. 4) and fixed effects (spec. 5) yield similar results, but both differ sharply from estimation using NLS (spec. 2 or 3), notably for the distance coefficient. Why? As just discussed, A-vW's NLS procedure is highly sensitive to the measurement of internal distances for the multilateral resistance terms and ignoring that the intercept (in theory) equals -ln Y^T. Is there a way to compare the estimation results of A-vW and our approach *excluding* the measurement and potential specification errors?

In this section, we employ a Monte Carlo approach to show that our OLS method yields estimates of border and distance coefficient estimates that are virtually identical to those using A-vW's NLS method when we know the "true" model. To do this, in section 4.2.1 we construct the "true" bilateral international trade flows among 41 regions using the theoretical model of A-vW described in section 2. We assume the world is described precisely by equations (11) and (12), assuming various arbitrary values for α, ρ, and σ under alternative scenarios. Using Canadian-U.S. province and state data on GDPs and bilateral distances and dummy variables for borders, we can compute the *true* bilateral trade flows *and true* multilateral resistance terms associated with these economic characteristics for given values of parameters α, ρ, and σ. We then assume that there exists a log-normally distributed error term for each trade flow equation. We make 5,000 draws for each trade equation and run various regression specifications 5,000 times.[25] We will consider first two different sets of given parameter values and five specifications. We use *GAUSS* in all estimates. In section 4.2.2, to show that this approach works in the more traditional context of world trade flows, we employ the same Monte Carlo approach and provide the results.

4.2.1. Monte Carlo Analysis #1: Canada-U.S.

We consider five specifications. Specification (1) is the basic gravity model ignoring multilateral resistance terms, as used by McCallum. The specification is analogous to equation (11) excluding the MR terms. In the context of the theory, we should get biased estimates of the true parameters since we intentionally omit the true multilateral price terms or fixed effects. Specification (2) is the basic gravity model augmented with "atheoretical remoteness"

[17] Our Taylor-series expansion illustrates that the intercept also reflects world resistance and the dispersion of world income. We note that Balistreri and Hillbery (2004) addressed other concerns about the A-vW study as well, including A-vW's exclusion of interstate trade flows and their imposing symmetry on U.S.-Canadian border effects. Due to space limitations, we do not address these issues.

[25] The error terms' distribution is such that the R^2 (and standard error of the estimate) from a regression of trade on GDP, distance, and borders using a standard gravity equation is similar to that typically found (an R^2 of 0.7 to 0.8).

terms (**REMOTE**$_i$ and **REMOTE**$_j$), as in McCallum (1995), Helliwell (1996, 1997, 1998), and Wei (1996). Equation (11) would include **REMOTE**$_i$ and **REMOTE**$_j$, instead of P_i and P_j, where **REMOTE**$_i$ = $\ln\Sigma_j^N$(**DIS**$_{ij}$/**GDP**$_j$) and analogously for **REMOTE**$_j$. In the context of the theory, we should get biased estimates of the true parameters since we are using atheoretical measures of remoteness. This specification also ignores other multilateral trade costs. For Specification (3), we take the system of equations described in equations (12) to generate the "true" multilateral resistance terms associated with given values of $-\rho(\sigma$-1) and $-\alpha(\sigma$-1). We then estimate the regression (11) using the true values of the multilateral resistance terms. In the presence of the true MR terms, we expect the coefficient estimates to be virtually identical to the *true* parameters. Specification (4) uses region-specific fixed effects. As discussed earlier, region-specific fixed effects should also generate unbiased estimates of the coefficients. Specification (5) is our OLS equation (28). If our hypothesis is correct, the parameter estimates should be virtually identical to those estimated using Specifications 3 and 4.

Initially, we run these five specifications for two different scenarios of values for $a_1 = -\rho(\sigma-1)$ and $a_2 = -\alpha(\sigma-1)$. In both cases, we report three statistics. First, we report the average coefficient estimates for a_1 and a_2 from the 5,000 regressions for each specification. Second, we report the standard deviation of these 5,000 estimates. In the last column, we report the fraction of times (from the 5,000 regressions) that the coefficient estimate for a variable was within two standard errors of the true coefficient estimate.[26] All estimation was done using *GAUSS*.

Scenario 1. Assume $-\rho(\sigma-1) = -0.79$ and $-\alpha(\sigma-1) = -1.65$

For Scenario 1, we use the actual coefficient estimates found in A-vW using their two-country model. Table 2a reports the estimated values for the five specifications under this scenario in columns (2)-(4). There are two major results worth noting. First, the first two specifications provide biased estimates of the border and distance coefficient estimates, as expected. Second, both fixed effects and BV-OLS provide estimates very close to those using Specification 3, as expected. While the average BV-OLS coefficient estimates depart slightly from the average A-vW estimates, 98 percent of the border and distance (coefficient) estimates are within two standard errors of true values.

Scenario 2. Assume $-\rho(\sigma-1) = -1.25$ and $-\alpha(\sigma-1) = -1.54$

Now we choose values for $-\rho(\sigma-1)$ and $-\alpha(\sigma-1)$ that are identical to those estimated using fixed effects in Table 1. Table 2b provides the same set of information as in column Table 2a, but for this alternative set of true values. The results are robust to this alternative set of parameters. The BV-OLS coefficient estimates are within two standard errors of the true values 99 percent of the time.

Sensitivity Analysis: Varying $-\rho(\sigma-1)$ and $-\alpha(\sigma-1)$ each between -0.25 and -2.00

Given the success of these results, we decided to perform these simulations for a wide range of arbitrary

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[26]Note that the standard deviation refers to the square root of the variance of all the coefficient estimates for a specification. We also calculated the standard errors of each coefficient estimate. The last column in each table refers to the fraction of the 5,000 regressions that the estimated coefficient is within two standard errors of the true value.

values of the parameters. We considered a range for each variable's "true" coefficient from -0.25 to -2.00. Because of the large number of simulations, we used 1,000 runs per parameter pair. We basically found the same findings. First, regardless of the true values of the Border and Distance coefficients, the OLS Border coefficient estimate is within two standard errors of the true value no less than *93 percent* of the time. Second, the OLS Distance coefficient estimate is also within two standard errors of the true value no less than 93 percent of the time. For brevity, these results are not reported individually; however, they are available on request.

4.2.2. Monte Carlo Analysis #2: Gravity Equations for World Trade Flows

Of course, the gravity equation has been used over the past four decades to analyze economic and political determinants of a wide range of aggregate "flows." However, the most common usage of the gravity equation has been for explaining *world* (intra- and inter-continental) bilateral trade flows. The issues raised in A-vW (2003) and in this paper have potential relevance for the estimation of the effects of free trade agreements and of tariff rates on world trade flows. In the spirit of "generalizing" our technique to other contexts, we offer another sensitivity analysis.

In this section, we construct an set of "artificial" aggregate bilateral world trade flows among 88 countries for which data on the exogenous RHS variables discussed above were readily available.[27] Three exogenous RHS variables that typically explain world trade flows are countries' GDPs, their bilateral distances, and a dummy representing the presence (0) or absence (1) of a common land border ("NoAdjacency"). We then estimate the relationship among bilateral trade flows, national incomes, bilateral distances and NoAdjacency among 88 countries using our OLS method. We simply redo section 4.2.1's Monte Carlo simulations.[28]

We start with the system of equations (11) and (12), modified to 88 regions. Initially, we assigned two sets of possible parameters for $-\alpha(\sigma-1)$ and $-\rho(\sigma-1)$, the same two sets of values used for Table 2. We then calculated the "true" MR terms and "true" trade flows using equations (11) and (12). We then assume there exists a log-normally distributed error term. We make 1,000 draws for the equation and run various specifications 1,000 times.

For the world data set, the countries are chosen according to data availability and include the largest of the world's economies. GDPs in thousands of U.S. dollars are from the World Bank's *World Development Indicators*. Bilateral distances were calculated using the standard formula for geodesic, or "great circle," distances (http://mathworld.wolfram.com/GreatCircle.html). NoAdjacency is a dummy variable defined as 0 (1) if the two

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[27]The 88 countries are Argentina, Australia, Austria, Bangladesh, Belgium, Bolivia, Brazil, Bulgaria, Canada, Chile, China, Colombia, Costa Rica, Cote d'Ivoire, Cyprus, Denmark, Dominican Republic, Ecuador, Egypt, El Salvador, Finland, France, The Gambia, Germany, Ghana, Greece, Guatemala, Guinea-Bissau, Guyana, Haiti, Honduras, Hong Kong, Hungary, India, Indonesia, Iran, Ireland, Israel, Italy, Jamaica, Japan, Kenya, South Korea, Madagascar, Malawi, Malaysia, Mali, Mauritania, Mauritius, Mexico, Morocco, Mozambique, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Pakistan, Panama, Paraguay, Peru, Philippines, Poland, Portugal, Romania, Saudi Arabia, Senegal, Sierra Leone, Singapore, Spain, Sri Lanka, Sudan, Sweden, Switzerland, Syria, Thailand, Trinidad and Tobago, Tunisia, Turkey, Uganda, United Kingdom, United States, Uruguay, Venezuela, Zaire, Zambia, and Zimbabwe.

[28]Naturally, we could also introduce in this exercise an array of other typical bilateral dummies, such as common language, common EIA, etc. However, this would have no bearing on the generality of our results.

countries actually share (do not share) a common land border. In the typical gravity equation for world trade flows, adjacency is expected to augment trade; hence, NoAdjacency (like Border in the previous section) has an expected negative relationship with trade.

The notable finding is that the estimation biases for world trade flows are very small, and are *even smaller* than those found using OLS for the intra-continental (Canadian-U.S.) trade flow specifications. For example, consider the results for $-\alpha(\sigma-1) = -1.65$ and $-\rho(\sigma-1) = -0.79$. For U.S.-Canadian trade, the average Border estimation bias is 0.42 percent and the fraction of times the estimate is within two standard errors of the true value is 0.985. The average Distance estimation bias is 1.52 percent and the fraction of times the estimate is within two standard errors of the true value is 0.978. However, for world trade, the average Border estimation bias is 0.18 percent and the fraction of times the estimate is within two standard errors of the true value is 0.992. The average Distance estimation bias is 0.13 percent and the fraction of times the estimate is within two standard errors of the true value is 0.996. The results for $-\alpha(\sigma-1) = -1.54$ and $-\rho(\sigma-1) = -1.25$ are similar. In a sensitivity analysis, we have found that the small estimation bias is systematic. In fact, 79.4 percent of the estimation biases are smaller for world trade flows compared with intra-continental trade flows (although the two "border" variables have different economic interpretations). The distance variable is measured in the same manner for both data sets. For *all* parameter values for the distance variables' coefficients, the estimation bias for world trade flows is less than that for regional trade flows.[29]

These findings provide quantitative support to our hypothesis that our OLS method is not only a good approximation to NLS, but that it works even more effectively in the context in which it is most often used – the analysis of *global* trade flows.

4.3. Potential Uses of the Approximation Method

A question may surface about the potential relevance of estimating equation (28) in light of the alternative of fixed effects. If fixed effects yield consistent estimates of gravity-equation parameters, what additional benefit arises from the linear approximation of the MR terms and/or estimation of equation (28) using OLS? In the Canadian-U.S. "border-puzzle" cross-section context – or even the more typical gravity-equation analyses of international trade flows – our approximation approach might only provide more "transparency" about understanding the role of MR terms; equation (27) does simplify further the "significantly simplified" gravity model of equations (12) and (13) in A-vW (2003, p. 176). However, one may argue (as our editor did!) that fixed effects allow consistent estimation of cross-section gravity-equation parameters and is easier than estimating equation (28). Once one obtains consistent estimates of the gravity-equation parameters using fixed effects, is it all that computationally burdensome to run a system of 41 or 88 nonlinear equations to estimate the comparative statics?

Although these are valid questions, we suggest (at least) four potential uses of our approximation method that neither the fixed effects nor the A-vW NLS estimation technique can address. First, in cross-sectional analyses, the use of region- (or country-) specific fixed effects precludes including any region-specific explanatory variables

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[29]The systematically lower estimation bias for the distance coefficients for world relative to regional trade flows is related to the notion of multilateral economic densities, which we address in section 5.

that may be of interest to the research. For instance, the levels of foreign aid, domestic populations, infrastructure levels are all region specific. By including the MR approximation, this allows explicit inclusion of region-specific explanatory variables.

Second, while the context of this paper and A-vW is cross-sectional analysis for a given year, gravity equations are being applied increasingly to *panel data*, with both large cross-sectional and long time-series variation (45 years of annual data and increasing). Estimation of gravity equations using country-specific fixed effects to capture the time-varying MR terms for each country in a panel of 200 countries with 45 years would require 8999 (= (200x45) - 1) dummy variables, which becomes computationally burdensome. Some studies using "huge" panel data sets find the numbers of necessary dummy variables infeasible using "standard computer hardware." Alternatively, one can use the linear approximation method and estimate equation (28) using the panel where the *time-varying* relevant MR terms are included explicitly. Three recent applications of our approach in panel contexts are Egger and Nelson (2007), Nelson and Juhasz Silva (2007), and Melitz (2007), for which our OLS method worked successfully.

Third, recent empirical economic and political science research on the determinants of bilateral or regional international economic integration agreements (EIAs) has used probit models to estimate empirically the explanatory role of economic and/or political variables for the likelihood of an EIA between a pair of countries, cf., Mansfield and Reinhardt (2003), Baier and Bergstrand (2004), and Mansfield, Milner, and Pevehouse (2005). For instance, Baier and Bergstrand (2004) examined the role for country pairs' economic determinants of free trade agreements (FTAs). Among other results, they showed theoretically that the welfare of the two countries' representative consumers improved from a regional FTA the more "remote" the two countries were from the rest of the world (e.g., the Australia-New Zealand FTA). However, in theory this remoteness is economically the *MR terms* we have been addressing; the higher the MR terms for a country pair the more they benefit from bilateral trade, and the greater the welfare improvement from a regional FTA. However, like the earlier gravity equation literature, they measured empirically these MR terms using the *atheoretical remoteness* variables used by McCallum, Helliwell, Wei and others discussed earlier. However, as explanatory variables in a probit regression, the MR terms suggested by our linear approximation would provide instead *theoretically-motivated* MR measures.

Fourth, econometric analysis of the *ex post* effects of EIAs on bilateral trade flows has typically been conducted using cross-section gravity equations and OLS; such a method is a parametric approach. However, more recently a few authors have been investigating – using *nonparametric* methods – the effects of EIAs on trade flows, employing econometric considerations more common to labor econometrics, cf., Baier and Bergstrand (2006) and Egger, Egger and Greenaway (2007). For instance, Baier and Bergstrand used a (nonparametric) "matching" estimator to generate *ex post* effects of EIAs on country pairs' trade flows, where country pairs with and without EIAs were sorted according to "balancing properties" (i.e., where the distributions of the economic determinants of trade, such as GDPs, bilateral distances, etc., were the "identical"). In order to estimate the effects of EIAs, a necessary variable to address to secure "balancing" was a measure of multilateral resistance. The linear approximation approach provided a theoretically-motivated variable to capture the important role of MR terms in order to estimate nonparametrically (using the matching estimator) the effects of EIAs on trade, and secured the balancing properties.

Thus, the linear approximation approach has (at least) four potential uses outside the cross-sectional gravity-equation contexts described earlier and in A-vW.

5. Comparative Statics: ***When*** Does the Approximation Method Work Well, and ***Why***

The final test of the potential usefulness of the approximation approach is to determine when it works well for conducting comparative statics, and why. In section 5.1, we compute the comparative statics *analytically* and provide intuition for why the approach provides a "good" approximation of the comparative-static (overall) *country* effects for Canada and the United States provided in A-vW (2003). Yet, MR terms derived from first-order linear approximations are not likely to provide very precise estimates of *region-pair-specific* (such as Alberta-Alabama) comparative statics in the context of the Canadian-U.S. border-puzzle context, and we discuss why. In section 5.2, we move to other contexts, in particular the most common context – gravity equations of international trade flows among large numbers of countries – to examine under what conditions the approximation method works well for comparative statics – and when it does not – providing the first estimates of the effects of FTAs on international trade flows using the A-vW technique as well as our approximation method. We find that the approximation method works best (for comparative statics) the smaller the comparative-static effect, as would be expected from any linear Taylor-series expansion of a nonlinear equation; the further the deviation from the "center" the greater the approximation error, cf., Judd (1998, Ch. 6). However, slightly different from the emphasis in A-vW, the effects of trade costs on multilateral price terms are not *necessarily* the greatest for the smallest countries (with consequently large trading partners); instead we find that the effects are the largest for small countries *that are close* in distance. In section 5.3, we extend A-vW to show analytically why small, *close* countries have the largest changes in multilateral price terms. The complexity of the issue requires us to demonstrate this in two parts, an analytical proof and a simulation. Moreover, we show that economic size *relative to* bilateral distance can explain readily the approximation errors of the comparative statics. In other words, as equation (8) or (9) suggests, the key economic variable to explain differences in comparative statics across country pairs – and the approximation errors – is $\theta_j / t_{ij}^{\sigma-1}$.

5.1. Analytical Estimates of Country-Specific Comparative Statics using the Approximation Approach

Consistent estimates of the gravity equation coefficients (and the average border effect) can be obtained estimating a gravity equation adding region-specific fixed effects. However, as A-vW note, one still needs to use the coefficient estimates from OLS with fixed-effects along with the nonlinear system of equations (12) to generate the country-specific border effects. By contrast, our procedure allows one to estimate the country-specific border effects *without* employing the nonlinear system of equations. We now demonstrate this.

Recall equation (13) to calculate (region-specific) border effects for $\mathbf{x}_{ij}$, using its log-linear form:

$$\mathbf{BB}_{ij} = \ln\mathbf{x}_{ij} - \ln\mathbf{x}_{ij}^* = a_2 - \ln\mathbf{P}_i^{1-\sigma} + \ln\mathbf{P}_i^{*1-\sigma} - \ln\mathbf{P}_j^{1-\sigma} + \ln\mathbf{P}_j^{*1-\sigma} \qquad (33)$$

where $\mathbf{x}_{ij} = \mathbf{X}_{ij} / \mathbf{Y}_i\mathbf{Y}_j$, a_2 is the estimate of $-\alpha(\sigma-1)$, and $a_2 < 0$. We substitute equation (10) into equations (24) and (25) to find the MR terms with *and* without national borders. Substituting these results into equation (33) yields:

$$\mathbf{BB}_{ij} = \ln\mathbf{x}_{ij} - \ln\mathbf{x}_{ij}^* = a_2\left\{\left[1 - \left(\sum_{j=1}^{N}\theta_j\mathbf{BORDER}_{ij}\right) - \left(\sum_{i=1}^{N}\theta_j\mathbf{BORDER}_{ij}\right) + \left(\sum_{i=1}^{N}\sum_{j=1}^{N}\theta_i\theta_j\mathbf{BORDER}_{ij}\right)\right]\right\} \tag{34}$$

where $\mathbf{BORDER}_{ij}$ = 1 if regions i and j are not in the same nation and 0 otherwise and the distance components of the multilateral price terms cancel out. Thus, estimates of the comparative static border barriers do not require estimating the $P_i^{1-\sigma}$, $P_i{*}^{1-\sigma}$, $P_j^{1-\sigma}$, and $P_j{*}^{1-\sigma}$ terms using a custom nonlinear program.

While we can easily compute these terms using a computer, we can show that the country-specific effects for the Canadian-U.S. data can be readily computed analytically. For the simple Canadian-U.S. case, equation (34) can be calculated analytically once we have data on Canadian province and U.S. state GDPs and an estimate of a_2; we use a_2 = -1.65. Given the definition of $\mathbf{BORDER}_{ij}$, it turns out that the second term in the large brackets on the RHS in equation (34) is simply Canada's share of Canadian and U.S. GDPs (θ_{CA}= 0.07) and the third term in the brackets is simply the U.S. share of Canadian and U.S. GDPs (θ_{US}= 0.93). Consequently, the sum of these terms cancel out the 1 and the effect is -1.65 times the last term in the brackets. The last term simplifies to 2 θ_{CA} θ_{US}, or 0.13. Hence, the general equilibrium comparative static effect of the national border on the trade between a Canadian province and U.S. state, using our approximation method, is -1.65x0.13 = -0.21, implying that the ratio of trade with the barrier (BB) to trade *without* the barrier (NB) is 0.81 (= $e^{-0.21}$). This is larger than the A-vW multi-country estimate of 0.56. However, using simple weights (rather than GDP-share weights) our approximation method generates a comparative static effect of 0.54, virtually identical to the A-vW estimate.

5.1.1. A-vW's Implication 1

The intuition is similar to A-vW (2003, section II). As in A-vW's Implication 1, trade barriers reduce size-adjusted trade between large countries more than between small countries. Using the notation just introduced, equation (34) can be rewritten as:

$$\mathbf{BB}_{CA,US} = \ln\mathbf{x}_{ij} - \ln\mathbf{x}_{ij}^* = a_2\,[1 - \theta_{CA} - \theta_{US} + 2\,\theta_{CA}\,\theta_{US}] \tag{35}$$

When θ_{CA} (θ_{US}) is a fraction, $2\theta_{CA}\,\theta_{US} = 1 - \theta_{CA}^2 - \theta_{US}^2$. Hence, equation (35) can be rewritten as:

$$\mathbf{BB}_{CA,US} = \ln\mathbf{x}_{ij} - \ln\mathbf{x}_{ij}^* = a_2\,[1 - \theta_{CA} - \theta_{US} + 1 - \theta_{CA}^2 - \theta_{US}^2] \tag{36}$$

In this case, as in A-vW (2003, p. 176-177), $\theta_{CA} = 1 - \theta_{US}$ and $\theta_{US} = 1 - \theta_{CA}$. Hence, (36) becomes:

$$\mathbf{BB}_{CA,US} = \ln\mathbf{x}_{ij} - \ln\mathbf{x}_{ij}^* = a_2\,[\theta_{US} + \theta_{CA} - \theta_{CA}^2 - \theta_{US}^2] \tag{37}$$

which is identical to equation (15) in A-vW (2003, p. 177). The implications discussed there follow.

5.1.2. A-vW's Implication 2

Similarly, A-vW's Implication 2 holds also. A national border increases size-adjusted trade within small countries more than within large countries. For instance, using our method, $\mathbf{BB}_{CA,CA}$ can be calculated as:

$$\mathbf{BB}_{CA,CA} = \ln\mathbf{x}_{ij} - \ln\mathbf{x}_{ij}^* = a_2 [0 - \theta_{US} - \theta_{US} + 2\, \theta_{CA}\, \theta_{US}] \quad (38)$$

Since $2\theta_{CA}\, \theta_{US} = 1 - \theta_{CA}^2 - \theta_{US}^2$ and $\theta_{CA} = 1 - \theta_{US}$ and $\theta_{US} = 1 - \theta_{CA}$, equation (38) can be rewritten as:

$$\mathbf{BB}_{CA,US} = \ln\mathbf{x}_{ij} - \ln\mathbf{x}_{ij}^* = a_2 [-1 + 2\theta_{CA} - \theta_{CA}^2 - \theta_{US}^2] \quad (39)$$

which is identical to equation (15) in A-vW (2003, p. 177). The implications discussed there follow.

Letting $a_2 = -1.65$, $\theta_{CA} = 0.07$, and $\theta_{US} = 0.93$, our method yields a border effect ratio of intra-national Canadian trade with a border to intra-national Canadian trade *without* a border of 17.92, larger than the A-vW multi-country estimate of 5.96. Using simple weights (rather than GDP-share weights) our approximation generates a comparative static effect of 6.60, much closer to the A-vW multi-country estimate for Canada.

5.1.3. A-vW's Implication 3

Finally, A-vW's Implication 3 follows from Implications 1 and 2. A national border increases intranational *relative to* international trade more the smaller is Canada and the larger is the United States. Letting $a_2 = -1.65$, $\theta_{CA} = 0.07$, and $\theta_{US} = 0.93$, our method yields a ratio of intra-national relative to international trade with a border to that *without* a border of 21.54, much larger than the A-vW multi-country estimate of 10.70. However, using simple weights our approximation generates a ratio of 12.13, closer to the A-vW estimates. In fact, our estimate of 12.13 is within the range of estimates recently reported in a sensitivity analysis by Balistreri and Hillberry (2007).

5.1.4. Limitations of the Approximation Method

While our approximation method can generate border-effect estimates close to those reported in the recent "border-puzzle" debate, a more demanding test of the method is to evaluate the (general equilibrium) comparative statics for specific pairs of regions. In this particular context, the method provides only a crude approximation, since θ_{CA} and θ_{US} are identical for every region-pair. Consequently, the "country-wide" border effects are identical to the region-pair border effects. However, using A-vW's NLS system, the region-pair border effects vary from 0.32 to 0.49 with an average of 0.41 (using the A-vW two-country technique). Consequently, for particular pairs of Canadian provinces and U.S. states, the method cannot capture the aspect of A-vW that regions within smaller countries face larger multilateral resistance than regions within larger countries.

5.2. Comparative Statics using World Trade Flows

A-vW motivated the importance of estimating appropriate comparative statics in the context of one specific case: McCallum's Canadian-U.S. "border puzzle." However, for nearly half a century, the gravity equation in international trade has been used *most commonly* to analyze bilateral aggregate international trade flows and – in particular – the effects of free trade agreements (FTAs) on such flows, cf., Frankel (1997). In this section, we analyze three representative gravity equation applications to illustrate that our approximation method works in the most common context for the gravity equation and to show when our approximation works well . . . and when it does not.

5.2.1. NAFTA

One of the most common empirical and policy contexts for applying the gravity equation is to analyze the effects of a particular economic integration agreement (EIA) on trade between pairs of countries; the most common type of EIA is a free trade agreement (FTA). The vast bulk of gravity equation studies since the early 1960s have estimated the average "treatment" effect of an EIA on trade using dummy variables and OLS, cf., Tinbergen (1962). However, as A-vW (2003) reminds us, the dummy variable's coefficient estimate provides only the "partial" effect, not the full general equilibrium comparative static effect.

In this section we use the same Monte Carlo approach used earlier for our 88-world (see Section 4.2). We calculated the true trade flows using the A-vW NLS specification including real GDPs, bilateral distance, an adjacency dummy, a language dummy, and a dummy variable representing the presence or absence of an EIA. To keep the approach similar to the literature, we define "NoEIA" as one if the EIA does not exist, and zero if it does; the ratios calculated are then interpreted similar to the effects of "border barriers" discussed earlier. We calculated the effects by pairs of countries of NoEIA using A-vW. We then calculated the same comparative statics using our (GDP-share-weighted) approximation method.

In the first scenario, we considered the effect of the North American Free Trade Agreement. Table 3 provides the results of the effect of "NoNAFTA" on the trade between the NAFTA members. Table 3 is organized as follows. Column 1 lists various country pairs (i,j) in NAFTA. Column 2 provides the partial effect on the two countries' bilateral trade of NoNAFTA; trade is reduced by 50 percent by eliminating the FTA between the countries. This value is exogenously assumed based upon evidence to date that (after accounting for endogeneity bias) the average (partial treatment) effect of an FTA on raising trade between two countries is about 100 percent, cf., Baier and Bergstrand (2007); hence, removing an FTA reduces trade by 50 percent. Columns 3 and 4 provide the estimates of how country i's and j's MR terms, respectively, rise as a result of NoNAFTA, computed using the A-vW NLS method. Columns 5 and 6 provide the corresponding estimates of how i's and j's MR terms rise, computed using our approximation method. Column 7 provides our estimate of the "world resistance" term change using our method. Columns 8 and 9 provide the total (full general equilibrium) effects of NoNAFTA on bilateral trade of i and j using the A-vW and our approximation methods, respectively.

Several points are worth noting. First, our example provides (one of) the first application(s) of the A-vW technique outside the context of the Canadian-U.S. data, using a data set of world trade flows (the most common gravity equation context for trade). The A-vW results highlight the importance of accounting for *multilateral resistance*. Most notably, the MR terms of the relatively smaller NAFTA members – Canada and Mexico – increase substantively, by 35 and 25 percent, respectively. Second, all of the approximation-method comparative static total effects are within 15 percent of the "true" values (i.e., where "true" denotes those computed using the A-vW method). It is important to note that – in the *absence* of estimating the structural price equations using A-vW – our approximation approach provides a *much more accurate* representation of the general equilibrium comparative static effects than simply using the coefficient estimate of the FTA dummy variable from a gravity equation (with or without fixed effects). The USA-Mexico approximation is within 3 percent of the true value, while the USA-Canada approximation is 8 percent lower and Canadian-Mexico's is 14 percent lower. Third, the approximation method distinguishes well between "small" and "large" countries. For instance, A-vW's method suggests that Mexico's MR term should increase by 25 percent, whereas the approximation method suggests a 19 percent increase. Fourth, the NAFTA case provides a ready first insight into where the approximation method will work poorly – trade between two countries that are small in economic size but fairly close in distance (here, on the same continent). We will see shortly that this is systematic in simulations, and can be explained economically. Naturally, since the comparative static effect for Canada-Mexico is the largest of the three effects, it has the largest approximation error, as standard to Taylor approximations.

In general equilibrium, bilateral trade among non-members of NAFTA, the vast bulk of the 3872 country pairs (88x88/2), are also affected because of changes in their multilateral resistance terms. However, these effects are small and for these 3869 non-NAFTA country pairs the approximation method yields comparative statics that are within 2 percent of the true values 95 percent of the time.

5.2.2. The European Economic Area

The most important economic integration agreement in post-WWII history has been European economic integration. Consequently, an important context to evaluate the approximation method's accuracy is measuring the trade-cost effects of removing the "European Economic Area," or "NoEEA." First, among our 88 countries, the potential number of country-pairs that are directly affected by EEA include 165 of the 3872 country-pairs in our sample. Reporting the results for all 165 pairs – much less the *other* 3707 pairs – is prohibitive in terms of space. Consequently, we can only summarize the results.

The most notable result from this Monte Carlo experiment is that *74 percent* of the comparative statics using the approximation method are within 5 percent of the "true" (A-vW-method-determined) comparative statics. Another 9 percent of the comparative statics using the approximation method have biases between 5 to 10 percent of the true values; hence, 83 percent have biases less than 10 percent. 92 percent of the approximation-method comparative statics are within 20 percent of the true values. As expected using a Taylor approximation, the largest biases for the country pairs with the largest changes in their MR terms (and hence in the comparative statics).

However, 8 percent of the approximation-method comparative statics differ from the true values by more than 20 percent. The largest error is 38 percent. Yet, *every single one* of the country-pairs with a bias greater than 20 percent includes either Austria, Belgium, Denmark, Ireland, or Switzerland in the pair. Moreover, every single pair where the approximation method performs poorly involves economically small EEA countries that are *close* to one another (and to large trading partners). Consistent with earlier findings, all these countries incur the *highest* increases in their MR terms from "NoEEA" because these countries are close to each other (and to other large trading partners).

5.2.3. All FTAs

We also conducted the same analysis for all FTAs in the 88-country sample. The main three findings from above hold in general. First, similar to the case of the EEA, 86 percent of pairs have an average bias of less than 20 percent. Second, the largest approximation errors are for the pairs of countries with the largest increases in their MR terms from having "NoFTA," as one would expect from a Taylor approximation. Third, the country pairs with the largest increases in their MR terms have *small GDPs* and are *close*, e.g., Uruguay-Paraguay in MERCOSUR, the Central American Common Market (CACM) countries, and the EEA countries discussed above.

5.2.4. Summary

We close this section noting the contrast between the results using our approximation versus using A-vW's method. Given the presence of nonlinearities, computing comparative static effects using A-vW's system of nonlinear equations is preferable. However, our approximation method provides a ready alternative method for estimating coefficients *and* for calculating (approximations of) *pair-specific* general equilibrium comparative statics. Moreover, recent work in Bergstrand, Egger and Larch (2007) indicates that obtaining solutions to systems of nonlinear price equations using A-vW may be difficult in contexts beyond the special Canadian-U.S. case with a single national border; in their paper, in the presence of asymmetric bilateral trade costs, solutions to systems of nonlinear equations using A-vW prove difficult.[30] We find in our general setting of world trade flows that our approximation method for computing pair-specific general equilibrium comparative statics is accurate with 10 percent of the "true"values in *83 percent* of our 3872 country pairings. This result – demanding only OLS – is clearly an improvement over simply using the coefficient estimate of an FTA dummy variable, as is typically done.

5.3. Explaining the Large MR Changes and the Approximation Errors

In this final section, we address two concerns. First, as with any Taylor expansion, the approximation errors will be largest for the largest changes relative to the center. Since Taylor expansions approximate better (generally) the higher the order, we discuss the factors likely influencing the approximation error, using a second-order Taylor expansion to illustrate them. Second, the Monte Carlo analysis above indicated that the largest MR term changes (from trade costs) were not necessarily for the economically smallest countries (with consequently large trading partners) as A-vW suggested, but rather small countries that are *physically close*. In this part, we present two results. First, we extend A-vW to show analytically in a world with *symmetric* (but positive) trade costs that small countries with large trading partners *relative to* trade costs will tend to have larger MR changes from changing trade costs. Second, because of limitations of the analytical proof, we then demonstrate a simple fixed-point iteration procedure that eliminates the approximation errors without having to use NLS estimation or a higher-order Taylor expansion (which, as for modern dynamic macroeconomic models, is very difficult and outside the paper's scope). We show

[30] Bergstrand, Egger and Larch (2007) find that applying the A-vW system of equations to standard contexts of world trade flows and asymmetric bilateral trade costs leads to solutions with complex numbers.

that the *sole* economic variable that explains differences in comparative statics across country pairs and their approximation errors is $\theta_j / t_{ij}^{\sigma-1}$; that is, the approximation errors are largest for small countries that are physically close (i.e., small $t_{ij}^{\sigma-1}$).

5.3.1. A Second-Order Taylor-Series Expansion

As documented above, the Taylor-series approximations of the MR terms are poorest when the true MR terms are large. As with any Taylor-series expansion, the approximation works best for small changes around the "center"; in our case, this is the average trade cost (t). Judd (1998) discusses the details and shows for a simple exponential function (centered at unity) that the "quality" of the approximation falls as the level of the variable moves further from unity. In general, higher-order Taylor-series expansions can provide better approximations. However, Judd (1998, pp. 197-8) provides an example that shows that the approximation error *can* still increase with the introduction of higher-order terms. It is important to note that the most commonly used expansion in modern dynamic macroeconomic models is still the first-order expansion, cf., Christiano, Eichenbaum, and Evans (2005). An alternative approach that may work better, but is beyond the scope of this paper, is a Padé approximation.

To understand the economic sources of the Taylor approximation errors, we first consider analytically a *second-order* Taylor-series expansion of equation (14), centered around a symmetric world (both trade costs and GDP shares). We report only the first set of derivations, akin to equation (20) in section 3:

$$
\begin{aligned}
& P^{1-\sigma} + (1-\sigma) P^{1-\sigma} (\ln P_{\mathrm{i}} - \ln P) \\
& \qquad + \frac{1}{2}(1-\sigma)^2 P^{1-\sigma} (\ln P_{\mathrm{i}} - \ln P)^2 \\
& = \sum_{\mathrm{j}=1}^{\mathrm{N}} \Big[\ \theta P^{-(1-\sigma)} t^{1-\sigma} - \left(\theta P^{-(1-\sigma)} t^{1-\sigma}\right)(1-\sigma)\left(\ln P_{\mathrm{j}} - \ln P\right) \\
& \qquad + \left(\theta P^{-(1-\sigma)} t^{1-\sigma}\right)\left(\ln \theta_{\mathrm{j}} - \ln \theta\right) + \left(\theta P^{-(1-\sigma)} t^{1-\sigma}\right)(1-\sigma)\left(\ln t_{\mathrm{ij}} - \ln t\right) \\
& - \frac{1}{2}(1-\sigma)^2 \left(\theta P^{-(1-\sigma)} t^{1-\sigma}\right)\left(\ln P_{\mathrm{j}} - \ln P\right)^2 \\
& \qquad + \frac{1}{2}\left(\theta P^{-(1-\sigma)} t^{1-\sigma}\right)\left(\ln \theta_{\mathrm{j}} - \ln \theta\right)^2 \\
& + \frac{1}{2}(1-\sigma)^2 \left(\theta P^{-(1-\sigma)} t^{1-\sigma}\right)\left(\ln t_{\mathrm{ij}} - \ln t\right)^2 \\
& - 2 \cdot \frac{1}{2}(1-\sigma)^2 \left(\theta P^{-(1-\sigma)} t^{1-\sigma}\right)\left(\ln P_{\mathrm{j}} - \ln P\right)\left(\ln \theta_{\mathrm{j}} - \ln \theta\right) \\
& - 2 \cdot \frac{1}{2}(1-\sigma)^2 \left(\theta P^{-(1-\sigma)} t^{1-\sigma}\right)\left(\ln P_{\mathrm{j}} - \ln P\right)\left(\ln t_{\mathrm{ij}} - \ln t\right) \\
& + 2 \cdot \frac{1}{2}(1-\sigma)^2 \left(\theta P^{-(1-\sigma)} t^{1-\sigma}\right)\left(\ln \theta_{\mathrm{j}} - \ln \theta\right)\left(\ln t_{\mathrm{ij}} - \ln t\right) \ \Big]
\end{aligned}
\tag{40}
$$

Clearly, this equation cannot be manipulated mathematically to solve for similar terms to the first-order expansion. A comparison of equation (40) with equation (20) shows that the RHS of the former equation includes three additional terms reflecting variances of the (endogenous) price term and of the (exogenous) GDP shares and trade costs, and three additional terms reflecting covariance among the (endogenous) price terms and (exogenous) GDP shares and trade costs. Thus, GDP shares *relative to* bilateral trade costs ($\theta_j / t_{ij}^{\sigma-1}$) play a critical role.

5.3.2. The Role of $\theta_j / t_{ij}^{\sigma-1}$ as the Source of Approximation Errors

Examining equation (8) or (9), it should come as no surprise that the key economic variable influencing outcomes is – not just economic size (θ_j) but – economic size *relative to* bilateral trade costs, $\theta_j / t_{ij}^{\sigma-1}$.[31] A-vW demonstrated clearly the importance of economic size for influencing MR terms; smaller countries have higher MR terms *ceteris paribus* and small countries' MR terms increase more for a given shock to trade costs. As A-vW (2003, p. 177) summarized, "For a small country trade is more important and trade barriers therefore have a bigger effect on multilateral resistance." Analogously, trade is more important for *close* countries and therefore border barriers should have a bigger impact on MR terms. In this section, we demonstrate two results. From an initial equilibrium of *symmetric* (but positive) trade costs, we show analytically that MR terms increase more for countries that are economically small *relative to* initial trade costs (t). In this regard, our proof is more general than A-vW's, which assumed an initial frictionless equilibrium. However, unlike A-vW, we cannot prove analytically that the change in MR for a given country (for an increase in trade costs) varies with the level of *pair-specific* trade costs (t_{ij}). To show this, we then turn to a "fixed-point" iteration analysis.

First, we show here that – from an initial equilibrium of symmetric positive trade costs ($t > 0$) – MR terms increase more (for a given increase in trade costs, dt) for countries that are small *relative to* initial trade costs (t). Differentiating (8) yields:

$$\begin{aligned}(1-\sigma) P_i^{-\sigma} dP_i &= \sum_{j=1}^{N} \theta_j t_{ij}^{1-\sigma} P_j^{\sigma-2} (-1)(1-\sigma) dP_i \\ &+ \sum_{j=1}^{N} \theta_j t_{ij}^{-\sigma} P_j^{\sigma-1} (1-\sigma) dt + \sum_{i=1}^{N} t_{ij}^{1-\sigma} P_j^{\sigma-1} d\theta_j\end{aligned} \tag{41}$$

Dividing by 1/(1-σ), setting $t_{ij} = t$ and $P_i = P_j = t^{1/2}$, and some algebraic manipulation yields:

[31] One might denote economic size relative to bilateral trade costs as "economic density." Since bilateral distance is a critical empirical variable influencing bilateral trade costs, this is consistent with the literature on "economic densities." Economic density refers, in general, to the amount of economic activity for a given physical area; a large literature exists on its measurement, cf., Ciccone and Hall (1996). In the trade context, a country's multilateral economic density is high when there is a strong negative correlation between partners' sizes and bilateral distances. For instance, Switzerland has a very multilateral economic density; its largest trading partners are quite close.

$$dP_i = (1/t^{1/2})[(1/2) - \theta_i + (1/2) \sum_{k=1}^{N} \theta_k^2]dt \tag{42}$$

Equation (42) confirms that for a given shock to trade costs (*dt*), MR terms increase more for small countries relative to *average* trade costs. However, this does not prove that small *and close* countries' MR terms increase more for a given trade-cost shock.

Given the limitations above of the second-order Taylor-series expansion and the analytical proof, we must turn to an alternative approach to identify the key economic variable that explains the errors. For this, we show that a simple "fixed-point" iteration on a matrix equation can generate precise (in our example, to seven decimal places) estimates of the "true" (or A-vW) MR terms. The key matrix in the equation is an NxN matrix of GDPs *scaled* by bilateral trade costs, $\theta_j / t_{ij}^{\sigma-1}$ (which we identified earlier as the key determinant of large comparative statics).[32]

We summarize the process briefly, referring the reader to Appendix A for technical details. First, calculate initial estimates of every $P_i^{1-\sigma}$ ($P_i^{*1-\sigma}$) using OLS, denoted $P_i^{1-\sigma}{}_0$ ($P_i^{*1-\sigma}{}_0$), for every region (i = 1, ..., N). Denote the Nx1 vector of these MR terms V_0 (V_0*) and the Nx1 vector of the inverses of each of these MR terms V_0^- (V_0^-*). Second, define an NxN matrix of GDP-share-weighted trade costs, *B*, where each element, b_{ij}, equals $\theta_j / t_{ij}^{\sigma-1}$. Third, compute V_{k+1} according to:

$$V_{k+1} = zBV_k^- + (1-z)V_k \tag{43}$$

starting at $k = 0$ until successive approximations are less than a predetermined value of ε (say, 1×10^{-9}), where $\varepsilon = \max|V_{k+1} - V_k|$ and *z* is a dampening factor with $z \in (0,1)$, and analogously for V_k*. Given the initial estimates of $P_i^{1-\sigma}$ ($P_i^{*1-\sigma}$) using OLS (i = 1, ..., N), this fixed-point iteration process will converge to the set of multilateral price terms identical to those generated using A-vW's NLS program. In the case where *B* has no dispersion in $\theta_j / t_{ij}^{\sigma-1}$, convergence will be virtually instantaneous.[33]

We have run this set of matrix calculations and the correlation coefficient between our MR terms (using fixed-point iteration) and A-vW's MR terms (using NLS) is 1.0 (reported to seven decimal places) in both the Canadian-U.S. context and the 88-country context. In the Canadian-U.S. case, convergence was achieved in 25 iterations (assuming $\rho(1-\sigma) = -0.79$ and $\alpha(1-\sigma) = -1.65$ for both cases, with *and* without the border) and the correlation of the multilateral resistance terms with the multilateral resistance terms constructed by A-vW is 1.0. Using parameter values of $\rho(1-\sigma) = -1.25$ and $\alpha(1-\sigma) = -1.54$, convergence is achieved after 21 iterations.

This method illustrates that the key *economic* variable explaining the approximation errors – as equation (8)

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[32] An advantage of the fixed-point method is that it is computationally much less resource-intensive than the nonlinear estimation technique used by A-vW, as it does not require computation of the Jacobian of the system of equations, nor does it even require that the inverse of the Jacobian *exists*.

[33] We use equation (10) to measure $t_{ij}^{\sigma-1}$ using bilateral distance and a coefficient estimate. The results are robust to alternative initial values of $P_i^{1-\sigma}$ ($P_i^{*1-\sigma}$).

would suggest – is GDP shares *relative to* bilateral trade costs, $\theta_j / t_{ij}^{\sigma-1}$.

6. Conclusions

Four years ago, theoretical foundations for the gravity equation in international trade were enhanced to recognize the *systematic bias* in coefficient estimates of bilateral trade-cost variables from omitting theoretically-motivated "multilateral (price) resistance" (MR) terms. Anderson and van Wincoop (2003) demonstrated that (i) consistent and efficient estimation of the bilateral gravity equation's coefficients in an N-region world required custom programming of a nonlinear system of trade and price equations, (ii) even if unbiased estimates of gravity equation coefficients could be obtained using fixed effects, general equilibrium comparative statics still required estimation of the full nonlinear system, and (iii) the model could be applied to resolve McCallum's "border puzzle."

This paper has attempted to make three potential contributions. First, we have demonstrated that a first-order log-linear Taylor series expansion of the nonlinear system of price equations suggests an alternative OLS log-linear specification that introduces theoretically-motivated *exogenous* MR terms. Second, empirical applications and Monte Carlo simulations suggest that the method yields virtually identical coefficient estimates to fixed effects and NLS estimation. Third, we have shown that the comparative statics associated with our approximation method have a bias no more than 5 percent in 74 percent of the 3872 country pairings of 88 countries examined. Moreover, we have identified the size of countries relative to their bilateral trade costs as the key economic variable explaining the approximation errors.

Future work in this direction might consider three issues. First, bilateral trade costs are not symmetric, and future work should examine the relevance of this issue. Second, we have used a standard Taylor-series expansion; however, a Padé approximation may yield better estimates. Third, especially for comparative statics, higher order terms matter, and future work should address their incorporation.

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TABLE 1
Estimation Results

Parameters	(1) OLS w/o MR Terms	(2) A-vW NLLS-2	(3) A-vW NLLS-3	(4) OLS with MR Terms	(5) Fixed Effects	(6) A-vW NLLS-2a	(7) A-vW NLLS-2b
$-\rho(\sigma$-1)	-1.06	-0.79	-0.82	-1.26	-1.25	-0.92	-1.15
	(0.04)	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.04)
$-\alpha(\sigma$-1)	-0.71	-1.65	-1.59	-1.53	-1.54	-1.65	-1.67
	(0.06)	(0.08)	(0.08)	(0.07)	(0.06)	(0.07)	(0.07)
Avg. Error Terms							
US-US	-0.21	0.06	0.06	-0.01	0.00	0.05	0.04
CA-CA	1.95	-0.17	-0.02	0.03	0.00	-0.22	-0.32
US-CA	0.00	-0.05	-0.04	0.01	0.00	-0.04	-0.02
R^2	0.42	n.a.	n.a.	0.52	0.66	n.a.	n.a.
No. of obs.	1,511	1,511	1,511	1,511	1,511	1,511	1,511

Numbers in parentheses are standard errors of the estimates.

TABLE 2a
Monte Carlo Simulations: Scenario 1
True Border Coefficient = -1.65
True Distance Coefficient = -0.79

Specification	Coefficient Estimate Average	Standard Deviation	Fraction within 2 Standard Errors of True Value
(1) McCallum			
Border	-0.789	0.026	0.000
Distance	-0.562	0.017	0.000
(2) OLS w/Atheoretical Remoteness Terms			
Border	-0.804	0.026	0.000
Distance	-0.541	0.019	0.000
(3) A-vW			
Border	-1.650	0.051	0.973
Distance	-0.789	0.034	0.950
(4) Fixed Effects			
Border	-1.650	0.033	0.967
Distance	-0.790	0.033	0.943
(5) OLS with MR Terms			
Border	-1.643	0.033	0.985
Distance	-0.802	0.020	0.978

TABLE 2b
Monte Carlo Simulations: Scenario 2
True Border Coefficient = -1.54
True Distance Coefficient = -1.25

Specification	Coefficient Estimate Average	Standard Deviation	Fraction within 2 Standard Errors of True Value
(1) McCallum			
Border	-0.655	0.025	0.000
Distance	-0.952	0.017	0.000
(2) OLS w/Atheoretical Remoteness Terms			
Border	-0.664	0.026	0.000
Distance	-0.940	0.019	0.000
(3) A-vW			
Border	-1.540	0.051	0.977
Distance	-1.250	0.034	0.950
(4) Fixed Effects			
Border	-1.540	0.033	0.988
Distance	-1.250	0.033	0.942
(5) OLS with MR Terms			
Border	-1.529	0.033	0.999
Distance	-1.276	0.021	0.996

TABLE 3
NAFTA Comparative Statics

(1) Country-Pair (i - j)	(2) Partial Effect	(3) A-vW MR Effect i	(4) A-vW MR Effect j	(5) B-B MR Effect i	(6) B-B MR Effect j	(7) B-B World Effect	(8) A-vW Total Effect	(9) B-B Total Effect
USA - Mexico	0.50	1.02	1.25	1.03	1.19	0.99	0.63	0.60
USA - Canada	0.50	1.02	1.35	1.03	1.19	0.99	0.68	0.60
Canada-Mexico	0.50	1.35	1.25	1.19	1.19	0.99	0.84	0.70

Appendix A
(NOT INTENDED FOR PUBLICATION)

The technique described in the paper, BV-OLS, yields virtually identical gravity equation coefficient estimates to those estimated using region-specific fixed effects (which are unbiased estimates). However, fixed effects cannot be used to generate general equilibrium comparative statics. Because BV-OLS yields linear approximations, it does not provide precise estimates of the region-specific multilateral resistance (MR) terms (with or without borders). However, one need not estimate the entire system of equations using custom nonlinear least squares to generate the exact same estimates of the MR terms as with A-vW's NLLS estimation. Given initial estimates of the MR terms using BV-OLS, a version of fixed-point iteration can be used to generate *identical* MR terms as under the NLLS technique, and fixed-point iteration is computationally much less intensive than the A-vW NLLS technique. In particular, even though the system of equations that determines the MR terms is non-linear, our fixed-point iteration method does not require computation of the Jacobian of the system of equations, nor does it require that the inverse of the Jacobian exists. We show that our approach requires nothing more than simple matrix manipulation in *STATA*, *GAUSS*, or any similar matrix programming language.

The approach can be calculated for MR terms with or without borders; for demonstration here, we assume borders are present. First, BV-OLS yields estimates of multilateral resistance terms $P_i^{1-\sigma}$ for i=1,..., N regions (with borders) based upon the log-linear approximation. Denote V_0 as the Nx1 vector of these $P_i^{1-\sigma}$ terms and V_0^- as the Nx1 vector of their inverses ($P_i^{\sigma-1}$). The functional equation we solve is $f(V) = V - BV^-$, where B is an NxN matrix of GDP-share-weights relative to bilateral trade costs where each element, b_{ij}, equals $\theta_j / t_{ij}^{\sigma-1}$, where t_{ij} are defined in section 2. Evaluated at the equilibrium values of the MR terms, V^E and V^{-E}, then $f(V^E) = V - BV^{-E} = 0$.

The fixed-point iteration method we use has essentially only two steps. First, use coefficient estimates from BV-OLS to construct the matrix B and use BV-OLS estimates of $P_i^{1-\sigma}$ ($P_i^{\sigma-1}$) to construct the initial value of V_0 (V_0^-). Second, compute V_{k+1} according to:

$$V_{k+1} = zBV_k^- + (1-z)V_k \tag{A1}$$

starting at $k = 0$ until successive approximations are less than a predetermined value (e.g., $1\text{x}10^{-9}$) of $\varepsilon = \max|V_{k+1} - V_k|$, where $\max|V_{k+1} - V_k|$ is the largest error approximation and z is a damping factor with $z \in (0,1)$. The estimated V_{k+1} satisfying this second step is *identical* to the V estimated using A-vW's custom NLLS estimation.

The remainder of this appendix proves in mathematical detail why this version of the fixed-point iteration converges to a solution. First, the standard approach for fixed-point iteration is to start with an initial guess V_0 and iterate on:

$$V_{k+1} = BV_k^- \tag{A2}$$

starting at k=0. The above equation converges as long as BV^- is a contraction map; that is, a necessary condition for a fixed-point iteration to converge is that – for each row of the Jacobian of BV^- – the sum of the absolute values of each element is less than unity, cf., Gerald and Wheatley (1990). This condition is unlikely to hold in general and it certainly does not hold for the McCallum-A-vW-Feenstra data. Even if it is a contraction map, it may not be the case that iterating induces convergence to the fixed point.

To see why this iteration process will not work in this context, consider a simple univariate mapping of:

$$v = (1/2)v^{-1} \tag{A3}$$

Trivially, the fixed point of this mapping is $v^E = 1/\sqrt{2}$. Clearly, the Jacobian satisfies the necessary condition for the fixed-point iteration to converge. However, with any initial guess of $v_0 \neq 1/\sqrt{2}$, the iteration produces a periodic cycle. For example, choose $v_0 = 2$ and the "solution" iterates between

$$v_i = \begin{cases} 1/4 & i\ odd \\ 2 & i\ even \end{cases}$$

and convergence does not obtain. To induce convergence in this system, we simply add a damping factor z (z = 0.5) and iterate on:

$$v_{k+1} = z(1/2)v_k^{-1} + (1-z)v_k \tag{A4}$$

With an initial estimate of $v_0 = 2$ for k=0, iterating on (A4) causes convergence of v to the true value (within ten decimal places) in three iterations.

Consequently, to induce convergence in our context, we introduce the damping factor z, where $z \in (0,1)$, and (A2) becomes:

$$V_{k+1} = zBV_k^{-} + (1-z)V_k \tag{A5}$$

Note this implies that $V_{k+1} = V_k - z\,f(V_k)$. For an initial guess in the range of V^E, the fixed-point iteration will converge to $f(V^E) = 0$ if z is contracting (since z is less than unity), cf., Nirenberg (1975). Thus, for the class of models discussed in A-vW the solution to the price terms can be obtained by fixed-point iteration with a damping factor of $z \in (0,1)$. Note how similar this is to the Gauss-Newton iteration scheme discussed in Judd (1998). Unlike the Gauss-Newton iteration, this procedure does not require computing the Jacobian or its inverse, if the latter exists.

We applied this procedure to the McCallum-A-vW-Feenstra Canadian-U.S. data set, using a stopping rule of $\varepsilon < 1\mathrm{x}10^{-9}$ for all elements of V. If we use the parameter values in A-vW of $\rho(1-\sigma) = -0.79$ and $\alpha(1-\sigma) = -1.65$, convergence is achieved after 25 iterations (for both cases, with *and* without the border) and the correlation of the multilateral resistance terms with the multilateral resistance terms constructed by A-vW is 1.0 (reported to seven decimal places). If we use the parameter values in BV-OLS of $\rho(1-\sigma) = -1.25$ and $\alpha(1-\sigma) = -1.54$, convergence is achieved after 21 iterations (for both cases, with and without the border) and the correlation of the multilateral resistance terms with the multilateral resistance terms constructed by the A-vW NLLS methodology is 1.0 (reported to seven decimal places). Given that this methodology replicates perfectly the MR terms calculated by A-vW, the comparative statics are identical to those reported by A-vW (2003, 187).