Session 1: The Theoretical Gravity Model

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Outline

1. Introduction
2. The Theoretical Gravity Model
   - Gravity with Gravitas
   - Empirical Implications of Gravity Theory
3. Summary
The basic gravity model provides a respectable place to start.

But if we look more closely, we will find that it has some unattractive implications from an economic point of view.

Doing some theory allows us to reformulate the gravity model in much more attractive way.
The Relative Price Problem

The Basic Gravity Model

\[ \log(X_{ij}) = b_0 + b_1 \log(Y_i) + b_2 \log(Y_j) + b_3 \log(d_{ij}) + e_{ij} \]

- Yesterday, we interpreted \( b_3 = -1 \) as indicating that a 1% increase in bilateral distance (transport costs) is associated with a 1% decrease in bilateral trade.
- In fact, this presents some serious problems in the context of a world with many countries.
The Relative Price Problem

The Basic Gravity Model

\[ \log(X_{ij}) = b_0 + b_1 \log(Y_i) + b_2 \log(Y_j) + b_3 \log(d_{ij}) + e_{ij} \]

- Do trade flows between i and j only depend on bilateral trade costs, without any adjustment for the level of trade costs prevailing on other routes?
- If trade costs fall between i and j, will trade flows with all other countries remain unchanged?
- If trade costs on all routes fall by the same proportion, will trade everywhere increase by the same proportion?
To try and fix these problems, it makes sense to go back to fundamentals.

The basic gravity model picks up some important empirical regularities, but has been posited without any explicit theoretical foundation.

If we add in some micro-foundations, hopefully we will be able to derive something that looks a lot like gravity, but deals with the relative cost problem.
A number of papers develop solid theoretical bases for the gravity model.

We will focus on the “gravity with gravitas” model set out by Jim Anderson and Eric Van Wincoop in the AER (2003) and JEL (2004).

It is treated by many (most?) applied trade researchers as the empirical baseline.

Build on it by all means, but ignore it at your peril!
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To derive the AvW model, we proceed as follows (see handout):

1. Set out a consumption side based on love of variety preferences;
2. Set out a production side with large group monopolistic competition;
3. Introduce trade costs, and relate domestic and foreign prices;
4. Impose some macro identities and aggregate to produce a gravity-like model.
The Theoretical Gravity Model

Consumption Side

Love of Variety Preferences

\[ U_i = \sum_{k=1}^{K} \left\{ \int_{V \in V_i^k} \left[ x_i^k (v) \right]^{1 - \frac{1}{\sigma_k}} dv \right\}^{\frac{1}{1 - \frac{1}{\sigma_k}}} \]

- C countries \((i)\), K sectors \((k)\), each with measure V varieties \((v)\).
- Intrasectoral (between varieties) elasticity of substitution \(\sigma_k\).
- Can easily be varied: one sector only, Cobb-Douglas first tier, etc.
- Utility increases with greater consumption of each variety, and consumption of more varieties.
Constant Markup Pricing

\[ p_i^k (v) = \left( \frac{\sigma_k}{\sigma_k - 1} \right) w a_i^k \]

- Each firm makes a unique variety under increasing returns to scale. Marginal cost \( a \) is constant by country-sector.
- With a very large number of firms, each one of them takes the overall price level as given.
- This allows us to shut down strategic interactions: all firms in a sector price at the same, constant markup over marginal cost.
The Theoretical Gravity Model

Introducing Trade Costs

International trade is costly: if I want one unit to arrive, I must ship $\tau_{ij} \geq 1$ units.

Thus, the price of a variety produced in one country and consumed in another is increased by the same factor over the price in the home country.

Think of it like an ad valorem tariff, or variable transport costs. No fixed costs of market entry.
The Theoretical Gravity Model

Aggregate and use Macro-Identities to get Gravity with Gravitas

The AvW Gravity Model

\[ X_{ij}^k = \frac{Y_i^k E_j^k}{Y^k} \left\{ \frac{\tau_{ij}^k}{\Pi_i^k P_j^k} \right\}^{1-\sigma_k} \]

\[ (\Pi_i^k)^{1-\sigma_k} = \sum_{j=1}^C \left\{ \frac{\tau_{ij}^k}{P_j^k} \right\}^{1-\sigma_k} \frac{E_j^k}{Y^k} \]

\[ (P_j^k)^{1-\sigma_k} = \sum_{j=1}^C \left\{ \frac{\tau_{ij}^k}{\Pi_i^k} \right\}^{1-\sigma_k} \frac{Y_i^k}{Y^k} \]

- Bilateral trade depends positively on the size of the importing and exporting countries, but negatively on trade costs.
- The two price indices capture the fact that it is relative trade barriers that matter.
The AvW Gravity Model

\[ X_{ij}^k = \frac{Y_i^k E_j^k}{Y^k} \left\{ \frac{\tau_{ij}^k}{\prod_i^k P_j^k} \right\}^{1-\sigma_k} \]

\[ (\prod_i^k)^{1-\sigma_k} = \sum_{j=1}^C \left\{ \frac{\tau_{ij}^k}{P_j^k} \right\}^{1-\sigma_k} \frac{E_j^k}{Y_j^k} \]

\[ (P_j^k)^{1-\sigma_k} = \sum_{i=1}^C \left\{ \frac{\tau_{ij}^k}{\prod_i^k} \right\}^{1-\sigma_k} \frac{Y_i^k}{Y_j^k} \]

- \( \prod_i^k \) is outward multilateral resistance.
- Exports from i to j depend on bilateral trade costs, but also on trade costs affecting i’s exports to all other markets.
The AvW Gravity Model

\[
X_{ij}^k = \frac{Y_i^k E_j^k}{Y^k} \left\{ \frac{\tau_{ij}^k}{\prod_i^k P_j^k} \right\}^{1-\sigma_k}
\]

\[
(\prod_i^k)^{1-\sigma_k} = \sum C_{j=1} \left\{ \frac{\tau_{ij}^k}{P_j^k} \right\}^{1-\sigma_k} \frac{E_j^k}{Y^k}
\]

\[
(P_j^k)^{1-\sigma_k} = \sum C_{i=1} \left\{ \frac{\tau_{ij}^k}{\prod_i^k} \right\}^{1-\sigma_k} \frac{Y_i^k}{Y^k}
\]

- \(P_j^k\) is inward multilateral resistance.
- Exports from i to j depend on bilateral trade costs, but also on trade costs affecting j’s imports from all other markets.
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AvW looks like gravity, but also has some important differences with respect to the basic model.

- Inclusion of the multilateral resistance terms.
- Selection of variables.
- Interaction between trade costs and the substitution elasticity.

What are the implications of these differences for empirical work?
The AvW Gravity Model

\[
X_{ij}^k = \frac{\gamma_i^k E_j^k}{\gamma^k} \left\{ \frac{\tau_{ij}^k}{\Pi_i^k P_j^k} \right\}^{1-\sigma_k} \\
(\Pi_i^k)^{1-\sigma_k} = \sum_{j=1}^C \left\{ \frac{\tau_{ij}^k}{P_j^k} \right\}^{1-\sigma_k} \frac{E_j^k}{\gamma^k}; \left( P_j^k \right)^{1-\sigma_k} = \sum_{i=1}^C \left\{ \frac{\tau_{ij}^k}{\Pi_i^k} \right\}^{1-\sigma_k} \frac{\gamma_i^k}{\gamma^k}
\]

- The AvW gravity model needs trade data in nominal value terms.
- Each observation should represent a unidirectional flow between a pair of countries, e.g. exports from i to j, not total trade in both directions (i to j or j to i), or the average, etc.
The AvW Gravity Model

\[ X_{ij}^k = \frac{Y_i^k E_j^k}{Y_j^k} \left\{ \frac{\tau_{ij}^k}{\Pi_i^k P_j^k} \right\}^{1-\sigma_k} \]

\[ \left( \prod_i \right)^{1-\sigma_k} = \sum_{j=1}^{C} \left\{ \frac{\tau_{ij}^k}{P_j^k} \right\}^{1-\sigma_k} \frac{E_j^k}{Y_j^k} \cdot \left( P_j^k \right)^{1-\sigma_k} = \sum_{i=1}^{C} \left\{ \frac{\tau_{ij}^k}{\Pi_i^k} \right\}^{1-\sigma_k} \frac{Y_i^k}{Y_j^k} \]

- GDP should be in nominal, not real, terms.
- When the model is estimated sector by sector, we would ideally like sectoral expenditures and production, not economy-wide GDP.
We need to do something to take account of multilateral resistance.

Standard price indices (CPI, PPI, etc.) are not aggregated in the way implied by theory, and so can at best be a poor proxy for the true (ideal) price indices we need.
The Trade Costs Function

The model depends on trade costs $\tau_{ij}^k$, but they are not directly observable as an aggregate. We need to build them up by specifying them in terms of observables.

Most commonly, the trade costs function is specified as follows:

$$\ln \left( \tau_{ij}^k \right) = t_1 \ln \left( d_{ij} \right) + t_2 \text{lang}_{ij} + t_3 \ln \left( 1 + \text{tariff}_{ij} \right) + t_4 \text{border}_{ij} + \ldots$$
The basic gravity model gets the first four terms approximately correct, but leaves out the last two. Unless the MR terms have zero correlation with exports and trade costs (or GDP), then we have omitted variables bias.
How serious is this OV bias empirically?

- Distance coefficient from the original gravity model without fixed effects = -1.277***.
- Distance coefficient from the fixed effects gravity model = -1.596***.

The difference between these estimates is statistically significant at the 1% level. Is it economically significant?
The problem is easier stated than solved, because there is no way to observe $\ln \Pi_i^k + \ln P_j^k$.

- We cannot just find some data, include it, and fix the OV bias!
The basic gravity model is fine as a starting point for an empirical analysis.

But gravity models now need to take account of multilateral resistance, i.e. “relative prices matter”.

Next session looks at the two most common ways of accounting for multilateral resistance in applied work.