

# More on Production

Short Course on CGE Modeling, University of the South Pacific

John Gilbert

Professor  
Department of Economics and Finance  
Jon M. Huntsman School of Business  
Utah State University  
jgilbert@usu.edu

July 23-27, 2012



# Introduction

- We have assumed that the only inputs into production are primary factors.
- In real economies, a significant amount of demand for goods comes from other industries, i.e., intermediate input use.
- We can handle this with some modifications to the production structure.
- It is also common (but not universal) to assume that firm's engage in joint production, producing differentiated goods for domestic and foreign markets. This is very much like Armington.

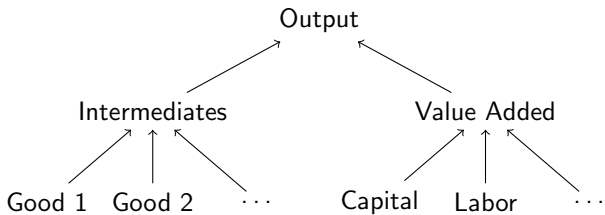
# Intermediates

- To introduce intermediates to model, we need to make three basic adjustments.
- First, we need to allow for intermediate use in the production functions.
- Next, we need to solve the GDP maximization problem to generate factor and intermediate demands.
- Finally, we need to incorporate intermediate input use in the market clearing conditions.

# Functional Forms

- The main issue in implementing intermediate goods in numerical models of production and trade relates not to the basic economics of the problem, but rather to the choice of functional form for the production function.
- We could use CES, but in most CGE models this approach is not adopted. The reason lies in the relatively restrictive assumptions of the CES form, of which Cobb-Douglas is a special case. In particular, this functional form assumes a common elasticity of substitution across inputs, implying that capital and labor are as substitutable with each other as, say, labor is with steel. This seems unlikely to be the case in practice.
- As a result it is common to use a 'nested' functional form.

# Nested Functions



# Fixed Proportions

- Fixed proportions (Leontief) technology is often used for intermediate goods.
- The min function is not differentiable. Hence, we can approximate it with a differentiable function (see next slide).
- Or, we can recognize that with Leontief technology inputs will be used in fixed proportions to value-added (hence the name) and proceed accordingly.

# Fixed Proportions

The production function will take the form:

$$q_i = \frac{\gamma_i}{1 - \sum_{\forall h \in \mathbf{I}} a_{hi}} \left[ \sum_{\forall j \in \mathbf{J}} \delta_{ji} F_{ji}^{\rho_i} \right]^{\frac{1}{\rho_i}} \quad \forall i \in \mathbf{I}$$

The net price equation is:

$$p_i^N = p_i - \sum_{\forall h \in \mathbf{I}} p_h a_{hi} \quad \forall i \in \mathbf{I}$$

and the factor demand equations are:

$$r_j = p_i^N q_i \left[ \sum_{\forall k \in \mathbf{J}} \delta_{ki} F_{ki}^{\rho_i} \right]^{-1} \delta_{ji} F_{ji}^{\rho_i - 1} \quad \forall j \in \mathbf{J}, \forall i \in \mathbf{I}$$

# Variable Proportions

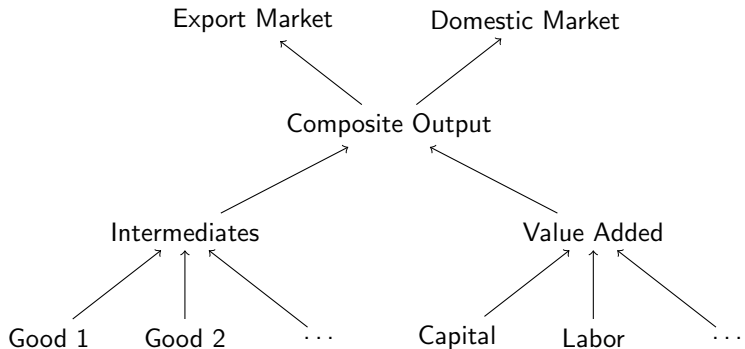
- If we want variable proportions in intermediate use, we can specify three different CES functions, one for output, one for value-added, and one for intermediate use.
- Factor and intermediate demands can then be derived as usual.
- This is not as messy as it might seem. Remember, in GAMS there is no particular advantage to compressing the expressions.
- We can differentiate each function and apply the chain rule.
- Calibration of each CES function is the same as previous examples.
- By varying the elasticities, we can approximate the fixed proportions case.



# Joint Production

- Joint production refers to a situation where firms produce more than one output at the same time.
- For example, in the chemical industry joint production can occur by the very nature of chemical transformations.
- In the dairy industry, the basic raw input gives rise to a number of different outputs, and it may make economic sense for a single firm to process and sell many of them.
- In many trade-oriented CGE models joint production occurs because firms are assumed to produce differentiated products for sale in domestic and foreign markets.
- This may arise from, for example, different regulations or standards that must be met in different countries, or different language requirements in labeling, and so on.
- The degree of differentiation is usually small.

# Joint Production



- The problem is essentially the same as Armington, but applied to the firm's optimization.
- The firm first chooses an optimal output mix (i.e., the combination of export and domestic sales that maximizes the value of a unit of composite sales).
- The firm then maximizes profit as usual.
- The CET function is generally used as the aggregator. This is just the same as the CES function except  $\rho > 1$ .

- GAMS modeling issues are discussed in more detail Gilbert and Tower (2012), chapters 10 and 24.